



Testing financing constraints on firm investment using variable capital[☆]

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Abstract

We consider a dynamic multifactor model of investment with financing imperfections, adjustment costs and fixed and variable capital. We use the model to derive a test of financing constraints based on a reduced form variable capital equation. Simulation results show that this test correctly identifies financially constrained firms even when the estimation of firms' investment opportunities is very noisy. In addition, the test is well specified in the presence of both concave and convex adjustment costs of fixed capital. We confirm empirically the validity of this test on a sample of small Italian manufacturing companies.

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1. Introduction

In order to explain the aggregate behavior of investment and production, it is necessary to understand the factors that affect investment at the firm level. Financing imperfections may prevent firms from accessing external finance, rendering firms unable to invest unless internal finance is available. It is therefore important to study the extent to which financing constraints matter for firms' investment decisions. This line of inquiry is also relevant for other areas of research, such as the literature on the role of internal capital markets and banks, as well as the macro literature on the financial accelerator.

Starting with Fazzari, Hubbard, and Petersen (1988), several studies investigate the presence of financing constraints by estimating the Q model of investment with cash flow included as an explanatory variable.¹ They argue informally that under certain conditions, and in the absence of financing frictions, Tobin's average Q is equal to marginal q , and is a sufficient statistic for firm investment (Hayashi, 1982). It follows that conditional on Q , cash flow should affect only the investment of financially constrained firms.

The motivation for this paper is that recent studies, starting with Kaplan and Zingales (1997, 2000), have shown that the correlation between fixed investment and cash flow is not a good indicator of the intensity of firm financing constraints. In particular, Erickson and Whited (2000) and Bond, Klemm, Newton-Smith, Syed, and Gertjan (2004) show that errors in measuring the expected profitability of firms explain most of the observed positive correlation between fixed investment and cash flow. Moreover, Gomes (2001), Pratap (2003), and Moyen (2004) simulate industries with heterogeneous firms that may face financing frictions. They show that the correlation between fixed investment and cash flow may be positive for financially unconstrained firms, and even larger than that of financially constrained firms.² Finally, Caballero and Leahy (1996) show that the failure of the investment-cash flow correlation as a measure of financing constraints may be caused not only by the measurement error in Q , but also by misspecification and omitted variable problems.

The objective of this paper is to develop a new financing constraints test that is robust to these problems and has the following properties: (i) it is able to detect both the presence and the intensity of financing constraints on firm investment; (ii) it is efficient even in the presence of large errors in the measurement of the productivity shock; (iii) it is well specified under a wide range of assumptions concerning the adjustment costs of fixed capital.

The test is derived from a structural model of a risk-neutral firm that generates output using two complementary factors of production, namely, fixed and variable capital. Fixed capital is irreversible, while variable capital can be adjusted without frictions. Because of an enforceability problem, the firm can obtain external financing only if it secures the funds with collateral. The assets of the firm can only be partially collateralizable and some down payment is needed to finance investment.

We describe the optimality conditions of the model and we demonstrate that under the hypothesis of financing imperfections, the correlation between financial wealth and variable capital investment is a reliable indicator of the presence of financing constraints.

¹See Hubbard (1998) for a review of this literature.

²Alti (2003) and Abel and Eberly (2001, 2005) develop theoretical frameworks in which positive investment-cash flow correlations arise in the absence of financial markets imperfections.

We use this result to develop a formal financing constraint test based on a reduced-form variable investment equation. This new test has two main advantages with respect to the previous literature. First, variable investment is less influenced by adjustment costs than fixed investment. This property reduces misspecification and omitted variable problems in the investment equation, thereby making it easier to distinguish the contribution of financial factors from the contribution of productivity shocks to firms' investment decisions. Second, while fixed investment decisions are forward looking, variable investment decisions are mostly affected by the current productivity shock, which is relatively easy to estimate even if only balance sheet data are available. Therefore, our financing constraints test does not require the estimation of Tobin's Q , and it can be applied also to small privately owned firms not quoted on the stock markets. This property of the test is important. The previous investment literature mainly studies the financing constraints of large firms quoted on the stock markets, even though financing frictions are mostly relevant for the financing of small privately owned firms.³ One reason for this bias is that the previous literature focuses mostly on the Q model, where average Q is computed as the ratio of the market value of the firm to the replacement value of its assets. However, because the market value is easily measurable only for publicly traded firms, this approach precludes the analysis of the effects of financing constraints on privately owned firms.⁴

We study the properties of the new financing constraints test by solving the model and simulating several industries with heterogeneous firms. We show that the sensitivity of variable capital to financial wealth is able to detect both the presence and the intensity of financing constraints on firm investment. This result is robust to both concave and convex adjustment costs of fixed capital. More importantly, large observational errors in measuring the productivity shock do not affect the power of the test, because the financial wealth of the simulated firms has a very low correlation with the current productivity shock.

We verify the validity of this test on two data sets of Italian manufacturing firms. These data sets are very useful for the purpose of this paper for two reasons: (i) almost all of the firms considered are small and not quoted on the stock market; (ii) all the firms are also covered by in-depth surveys with qualitative information about the financing problems the firms faced in funding investment.

We estimate the variable investment equation on these data sets and we confirm the predictions of the model. First, the estimated coefficients do not reject the restrictions imposed by the structural model. Second, the sensitivity of variable investment to internal finance is significantly positive for firms that are likely to face capital market imperfections

³Among the exceptions, Himmelberg and Petersen (1994) and Whited (2006) consider data sets of publicly owned firms, focusing explicitly on small and very small firms. Jaramillo, Schiantarelli, and Weiss (1996), Gelos and Werner (2002), and Lízal and Svejnar (2002) study samples of small privately owned firms in developing countries. However, the claim that small firms do not matter for developed economies, because large firms account for most of the aggregate employment and output, is not correct. For example, in 1995, small firms with less than 100 employees accounted for 37.9% of the total employment in the U.S. economy (source: U.S. Census).

⁴In principle, one can use other methods to calculate marginal q using only balance sheet data. For example, Gilchrist and Himmelberg (1998) apply the VAR approach of Abel and Blanchard (1986) to a panel of firms. But the resulting estimate of marginal q is probably even more noisy than the average Q calculated using the stock market valuation of firms, and hence the financing constraints test based on this measure of marginal q is probably even less reliable than the test based on average Q .

(according to the qualitative survey), while it is always very small and not significantly different from zero for other firms.

This paper contributes to both the theoretical and empirical literature on financing constraints and firm investment. The simulation results of this paper are related to [Gomes \(2001\)](#), [Pratap \(2003\)](#), and [Moyen \(2004\)](#). Because we consider both convex and nonconvex adjustment costs, we are able to clarify the link between adjustment costs and the investment-internal finance relation. In our benchmark model, fixed capital is irreversible and q is not a sufficient statistic for investment. In this case, cash flow-investment sensitivity is highest for financially unconstrained firms, even in the absence of measurement errors in q , as [Moyen \(2004\)](#) also finds. In the alternative model fixed capital is subject to convex adjustment costs and q is a sufficient statistic for investment. We show that in this case, cash flow-investment sensitivity is a reliable indicator of financing constraints, even in the presence of large measurement errors in q .

Because of this paper's emphasis on the importance of adjustment costs in explaining the investment decisions made by firms, it is related to [Barnett and Sakellaris \(1998\)](#) and [Abel and Eberly \(2002\)](#), who analyze the implications of different types of adjustment costs on the relation between marginal Q and investment at both the firm level and the aggregate level. Moreover, it is related to [Whited \(2006\)](#), who shows that in the presence of fixed costs of investment, constrained firms are less likely to undertake a new, large investment project than unconstrained firms, after controlling for expected productivity and the time elapsed since the last large investment project.

The empirical section of this paper uses a structural model of firm investment to derive a financing constraints test that is based on a simple reduced-form linear investment equation. A similar approach is followed by [Hennessy, Levy, and Whited \(2007\)](#), who derive an enhanced version of the Q model that allows for the presence of financing frictions and debt overhang. [Carpenter and Petersen \(2003\)](#) estimate a version of the Q model with cash flow where the dependent variable is the growth of total assets of the firm rather than the fixed investment rate.

Our method of testing for financing constraints on firm investment can be applied using any reversible factor of production. This paper considers the use of variable inputs as the dependent variable of the test. It is therefore also related to [Kashyap, Lamont, and Stein \(1994\)](#) and [Carpenter, Fazzari, and Petersen \(1998\)](#), who show that inventories at the firm level are very sensitive to internal finance, especially for those firms that a priori are more likely to be financially constrained. With respect to these two studies, our paper, in addition to proposing a more rigorous financing constraints test that identifies both the presence and the intensity of financing constraints, has two further advantages. First, while the flow of the use of materials is very close to a frictionless variable input, changes in total inventories are potentially subject to various adjustment costs, such as fixed costs that imply (S,s) type of inventory policies. Hence, the reduced-form linear inventory models estimated by [Kashyap, Lamont, and Stein \(1994\)](#) and [Carpenter, Fazzari, and Petersen \(1998\)](#) are potentially subject to misspecification problems, which make it difficult to distinguish whether internal finance significantly affects inventories because of financing frictions or because it is capturing other omitted information. Second, even if financing constraints affect inventory decisions, this does not necessarily imply that they also affect investment in production inputs and the firm's level of production. Indeed, the very fact that a financially constrained firm can absorb a reduction in cash flow with a reduction in inventories means that it may be able to maintain the desired flow of variable inputs in the

production process. Thus, the objective of this paper is precisely to estimate the intensity of financing constraints on the investment in variable inputs and in turn on the firm's production.

This paper is organized as follows. Section 2 describes the model. Section 3 defines the new financing constraints test. Section 4 illustrates the simulation results, and Sections 5 and 6 verify the validity of the new financing constraints test using a balanced panel of Italian firms. Finally, Section 7 summarizes and concludes.

2. The model

The aim of this section is to develop a structural model of investment with financing constraints and adjustment costs of fixed capital. We consider a risk-neutral firm whose objective is to maximize the discounted sum of future expected dividends. The discount factor is equal to $1/R$, where $R = 1 + r$ and r is the lending/borrowing risk-free interest rate.

The firm operates with two inputs, k_t and l_t , which denote fixed capital and variable capital, respectively. The production function is strictly concave in both factors. We assume a Cobb-Douglas functional form:

$$y_t = \theta_t k_t^\alpha l_t^\beta \quad \text{with } \alpha + \beta < 1. \quad (1)$$

All prices are constant and normalized to one. This simplifying assumption will be relaxed in the empirical section of the paper. The factor θ_t is a productivity shock that follows a stationary AR(1) stochastic process. For simplicity we assume that variable capital is nondurable and fully depreciates after one period, while fixed capital is durable and depreciates at the rate δ ,

$$0 < \delta < 1. \quad (2)$$

Moreover, variable capital investment is not subject to adjustment costs, while gross fixed capital investment, i_{t+1} , is irreversible, that is

$$i_{t+1} \geq 0, \quad (3)$$

and is given by

$$i_{t+1} \equiv k_{t+1} - (1 - \delta)k_t. \quad (4)$$

We assume full irreversibility for convenience, but the results of the paper would also hold for other types of nonconvex adjustment costs, such as partial irreversibility or fixed costs. In Section 5 we relax this assumption and allow for convex adjustment costs.

Financial imperfections are introduced by assuming that new share issues and risky debt are not available. At time t the firm can borrow one-period debt from or lend one-period debt to the banks at the market riskless rate r , where the face value of debt is denoted by b_{t+1} . A positive (negative) b_{t+1} indicates that the firm is a net borrower (lender). Banks only lend secured debt, and the only collateral they accept is physical capital. Therefore, at time t the borrowing capacity of the firm is limited by the following constraints:

$$b_{t+1} \leq \nu k_{t+1}, \quad (5)$$

$$d_t \geq 0, \quad (6)$$

and

$$0 < v \leq 1 - \delta,$$

where d_t are dividends and v is the share of fixed capital that can be used as collateral. One possible justification for constraint (5) is that the firm can hide the revenues from production. Given the banks are unable to observe such revenues, they can only claim the residual value of the firm's physical assets as repayment of the debt (Hart and Moore, 1998).⁵ If $v = 1 - \delta$, then all the residual value of fixed capital is accepted as collateral. This is possible because we assume that the irreversibility constraint (3) does not apply when the firm as a whole is liquidated and all its assets are sold.⁶

The timing of the model is as follows. New capital purchased in period $t - 1$ generates output in period t . At the beginning of period t the firm's technology becomes useless with exogenous probability $1 - \gamma$. In this case the assets of the firm are sold and the revenues are distributed as dividends. With probability γ , the firm continues activity. In this case θ_t is realized, y_t is produced using k_t and l_t (the production inputs purchased in the previous period), and b_t is repaid. The exogenous exit probability is necessary in order to generate simulated industries in which a fraction of firms are financially constrained in equilibrium. If $\gamma = 1$ and firms live forever, then they eventually accumulate enough wealth to become unconstrained, and the simulated industry always converges to a stationary distribution of financially unconstrained firms, no matter how tight the financing constraint (5) is.

It is useful to define the net worth of the firm w_t , after the debt b_t is repaid, as

$$w_t = w_t^F + (1 - \delta)k_t, \quad (7)$$

where w_t^F denotes financial wealth and is given by

$$w_t^F = y_t - b_t. \quad (8)$$

After producing, the firm allocates w_t^F plus the new borrowed funds between dividends, fixed capital investment, and variable capital investment according to the following budget constraint:

$$d_t + l_{t+1} + i_{t+1} = w_t^F + b_{t+1}/R. \quad (9)$$

For convenience, we define a_t as the stock of financial savings:

$$a_t \equiv -b_t.$$

We define a^* as the minimum level of financial savings such that the borrowing constraint (5) is never binding for every period $j \geq t$. The concavity of the production function (1) and the stationarity of the productivity shock θ ensure that a^* is positive and finite. Intuitively, when $a_t \geq a^*$ the returns from savings are always higher than the maximum losses from the production activity: $ra_t^* > \max_{k_t, \theta_t} R(l_{t+1} + i_{t+1} - y_t)$. Because the discount factor of the firm is equal to $1/R$, when $a_t < a^*$ the firm faces future expected financing constraints and always prefers to retain rather than to distribute earnings.

⁵Some authors argue that variable capital has a higher collateral value than fixed capital (Berger, Ofek, and Swary, 1996). Nevertheless, the results derived in this section are consistent with alternative specifications that allow for a positive collateral value of variable capital.

⁶In theory, the interactions between financing constraints and adjustment costs of fixed capital may imply that in some cases the firm is forced to liquidate the activity to repay the debt, even if it would be profitable to continue. In order to simplify the analysis, in this paper we focus on the set of parameters that do not allow this outcome to happen in equilibrium.

Alternatively, when $a_t \geq a^*$ the firm is indifferent between retaining and distributing net profits. Therefore, we make the following assumption:

Assumption 1. *If $a_t \geq a^*$, then the firm distributes net profits as dividends:*

$$d_t = y_t - l_{t+1} - i_{t+1} + \frac{ra_t}{R} \quad \text{if } a_t \geq a^*. \tag{10}$$

Eq. (10) implies that the firm distributes as dividends the extra savings above a^* . Assumption 1 is only necessary to provide a natural upper bound to the value of w_t^F , and it does not affect the real investment decisions of the firm.

Let us denote the firm’s value at time t , after θ_t is realized, by $V_t(w_t, \theta_t, k_t)$:

$$V_t(w_t, \theta_t, k_t) = \underset{k_{t+1}, l_{t+1}, b_{t+1}}{MAX} \pi_t + \frac{\gamma}{R} E_t[V_{t+1}(w_{t+1}, \theta_{t+1}, k_{t+1})], \tag{11}$$

$$\pi_t = \gamma d_t + (1 - \gamma)w_t. \tag{12}$$

The firm maximizes (11) subject to Eqs. (3), (5), (6) and (9). Appendix A provides a proof that the optimal policy functions $k_{t+1}(w_t, \theta_t, k_t)$, $l_{t+1}(w_t, \theta_t, k_t)$, and $b_{t+1}(w_t, \theta_t, k_t)$ exist and are unique.

In order to describe the optimality conditions of the model, we use Eq. (9) to substitute d_t in the value function (11). Let μ_t , λ_t , and ϕ_t be the Lagrangian multipliers associated, respectively, with the irreversibility constraint (3), the borrowing constraint (5), and the nonnegativity constraint on dividends (6). The solution of the problem is defined by the following conditions:

$$\phi_t = R\lambda_t + \gamma E_t(\phi_{t+1}), \tag{13}$$

$$E_t\left(\frac{\partial y_{t+1}}{\partial k_{t+1}}\right) = \{R[1 + E_t(\Psi_{t+1}^k)] - (1 - \delta)\} - R\mu_t + \Phi_t E_t(\mu_{t+1}), \tag{14}$$

$$E_t\left(\frac{\partial y_{t+1}}{\partial l_{t+1}}\right) = R[1 + E_t(\Psi_{t+1}^l)], \tag{15}$$

and

$$\left(1 - \frac{v}{R}\right)k_{t+1} + l_{t+1} \leq w_t^F + (1 - \delta)k_t - d_t, \tag{16}$$

where

$$\Phi_t \equiv \frac{\gamma(1 - \delta)}{1 + \gamma E_t(\phi_{t+1})}, \tag{17}$$

$$E_t(\Psi_{t+1}^k) \equiv \frac{(R - v)\lambda_t - \frac{\gamma}{R} cov\left(\phi_{t+1}, \frac{\partial y_{t+1}}{\partial k_{t+1}}\right)}{1 + \gamma E_t(\phi_{t+1})}, \tag{18}$$

and

$$E_t(\Psi_{t+1}^l) \equiv \frac{R\lambda_t - \frac{\gamma}{R} cov\left(\phi_{t+1}, \frac{\partial y_{t+1}}{\partial l_{t+1}}\right)}{1 + \gamma E_t(\phi_{t+1})}. \tag{19}$$

Eqs. (13)–(15) are the first-order conditions of b_{t+1} , k_{t+1} , and l_{t+1} , respectively. Eq. (16) combines the budget constraint (9) and the collateral constraint (5) and implies that the down payment necessary to buy k_{t+1} and l_{t+1} must be lower than the residual net worth after paying the dividends. By iterating forward Eq. (13), we obtain

$$\phi_t = R \sum_{j=0}^{\infty} \gamma^j E_t(\lambda_{t+j}). \quad (20)$$

Eq. (20) implies that as long as there are some current or future expected financing constraints, then $\phi_t > 0$ and the firm does not distribute dividends: $d_t = 0$. Eq. (14) represents the optimality condition for the fixed capital k_{t+1} . The left-hand side is the marginal productivity of fixed capital and the right-hand side the marginal cost of fixed capital. The term $\{R[1 + E_t(\Psi_{t+1}^k)] - (1 - \delta)\}$ is the shadow cost of buying one additional unit of fixed capital net of its residual value $(1 - \delta)$. The term $E_t(\Psi_{t+1}^k)$ is equal to zero if the firm is not financially constrained today or in the future. The terms μ_t and $E_t(\mu_{t+1})$ measure the shadow cost of a currently binding irreversibility constraint and of future *expected* irreversibility constraints, respectively. Eq. (15) is the optimality condition for the variable capital l_{t+1} . The term $E_t(\Psi_{t+1}^l)$ is directly related to λ_t , the Lagrange multiplier of the borrowing constraint (5).

If constraint (16) is not binding then $\lambda_t = 0$. In this case Eqs. (14) and (15) determine the optimal unconstrained capital levels k_{t+1}^u and l_{t+1}^u . If k_{t+1}^u is greater than $(1 - \delta)k_t$, then the irreversibility constraint (3) is not binding, the Lagrange multiplier μ_t is equal to zero, and $\{k_{t+1}^*, l_{t+1}^*\}$, the optimal investment choices, are determined by $\{k_{t+1}^u, l_{t+1}^u\}$. If k_{t+1}^u is smaller than $(1 - \delta)k_t$, then the irreversibility constraint is binding, k_{t+1} is constrained to be equal to $(1 - \delta)k_t$, and Eqs. (14) and (15) can be solved to determine l_{t+1}^{ic} and μ_t^{ic} . In this case the optimal investment choices $\{k_{t+1}^*, l_{t+1}^*\}$ are determined by $\{(1 - \delta)k_t, l_{t+1}^{ic}\}$. Alternatively, the collateral constraint is binding when financial wealth is not sufficient as a down payment for k_{t+1}^* and l_{t+1}^* , even if $d_t = 0$:

$$\left(1 - \frac{v}{R}\right)k_{t+1}^* + l_{t+1}^* > w_t + (1 - \delta)k_t. \quad (21)$$

In this case the constrained levels of capital k_{t+1}^c and l_{t+1}^c are such that

$$\left(1 - \frac{v}{R}\right)k_{t+1}^c + l_{t+1}^c = w_t + (1 - \delta)k_t, \quad (22)$$

and the solution is determined by the values k_{t+1}^c , l_{t+1}^c , λ_t , and μ_t that satisfy Eqs. (3), (14), (15), and (22).

3. A new test of financing constraints based on variable capital

One important property of variable capital is that Eq. (15) is not directly affected by the irreversibility constraint of fixed capital. The financing constraints test developed in this paper uses this property plus the fact that the term $E_t(\Psi_{t+1}^l)$, which summarizes the effect of financing constraints on variable capital investment, is a monotonously decreasing and convex function of $w_{i,t}^F$, as stated in the following proposition:

Proposition 1. *We define $w_t^{\max}(\theta_t, k_t)$ as the level of financial wealth such that the firm does not expect to be financially constrained now or in the future. It follows that for a given value*

of the state variables θ_t and k_t and for $w_t^F < w_t^{\max}$, $E_t(\Psi_{t+1}^l)$ is positive and is decreasing and convex in the amount of internal finance, that is,

$$\frac{\partial E_t(\Psi_{t+1}^l)}{\partial w_t^F} < 0, \quad \frac{\partial^2 E_t(\Psi_{t+1}^l)}{\partial (w_t^F)^2} > 0 \quad \text{and} \quad \lim_{w_t^F \rightarrow w_t^{MAX}} E_t(\Psi_{t+1}^l) = 0.$$

Conversely, if $w_t^F \geq w_t^{\max}$, then $E_t(\Psi_{t+1}^l) = 0$.

Proof. See Appendix B. \square

Proposition 1 applied to Eq. (15) establishes a link between financing imperfections and the real investment decisions of firms. It says that when a firm is financially constrained the availability of internal finance increases the investment in variable capital and reduces its marginal return. It is important to note that Proposition 1 cannot be applied to fixed capital investment because of the presence of the irreversibility constraint. If the irreversibility constraint is binding, then $k_{t+1} = (1 - \delta)k_t$ and $\mu_t > 0$. In this case a change in the intensity of financing constraints, which causes a change in $E_t(\Psi_{t+1}^k)$ in Eq. (14), affects the value of μ_t but does not affect fixed capital investment.

We therefore propose a new financing constraints test that applies Proposition 1 to variable capital investment decisions. If we take logs of both sides of Eq. (15) and solve for $\ln l_{t+1}$, we obtain

$$\ln l_{t+1} = \frac{1}{1 - \beta} \ln \frac{\beta}{R} + \frac{1}{1 - \beta} \ln E_t(\theta_{t+1}) + \frac{\alpha}{1 - \beta} \ln k_{t+1} - \frac{1}{1 - \beta} \ln [1 + E_t(\Psi_{t+1}^l)]. \quad (23)$$

Proposition 1 allows us to substitute $1 + E_t(\Psi_{t+1}^l)$ with a negative and convex function of w_t^F/w_t^{\max} . We approximate it as follows:

$$1 + E_t(\Psi_{t+1}^l) = (w_t^{\max}/w_t^F)^\eta, \quad (24)$$

where η is an indicator of the intensity of the financing constraints. The more the firm is financially constrained (in the model, this corresponds to a lower value of v , which tightens the financing constraints), the more the investment of the firm is sensitive to internal finance (meaning that $E_t(\Psi_{t+1}^l)$ increases more rapidly as w_t^F decreases) and the larger η is. The term w_t^{\max} is not observable in reality, but is itself a function of the other state variables. Intuitively, w_t^{\max} increases in $E_t(\theta_{t+1})$ because higher productivity increases the financing needs of the firm, and conditional on $E_t(\theta_{t+1})$ it decreases in k_t , because a larger existing stock of fixed capital implies that more financial wealth can be used to finance variable capital. Since k_{t+1} is highly correlated with k_t , our simulations show that a good approximation of w_t^{\max} is

$$w_t^{\max} = w^{\max} [E_t(\theta_{t+1})]^\zeta k_{t+1}^\omega. \quad (25)$$

Using Eqs. (24) and (25) in (23), and lagging Eq. (23) by one period, we obtain the following reduced-form variable capital equation:

$$\ln l_t = \pi_0 + \pi_1 \ln E_{t-1}(\theta_t) + \pi_2 \ln k_t + \pi_3 \ln w_{t-1}^F + \varepsilon_t, \quad (26)$$

$$\pi_0 \equiv \frac{1}{1 - \beta} \ln \left(\frac{\beta}{R} w^{\max} \right); \quad \pi_1 \equiv \frac{1 - \eta \zeta}{1 - \beta}; \quad \pi_2 \equiv \frac{\alpha - \eta \omega}{1 - \beta}; \quad \pi_3 \equiv \frac{\eta}{1 - \beta}. \quad (27)$$

The term ε_t includes the approximation errors. When estimating Eq. (26) with the empirical data it may also include measurement errors as well as unobservable productivity

shocks. Such problems are dealt with in the estimations in the empirical section of the paper.

The new financing constraints test is based on the coefficient π_3 . In the absence of financing frictions, η is equal to zero. This implies that $\pi_3 = 0$, $\pi_1 = \frac{1}{1-\beta}$, and $\pi_2 = \frac{\alpha}{1-\beta}$. Therefore, π_1 and π_2 can be used to recover the structural elasticities α and β . In the presence of financing constraints, η and π_3 are positive. The intuition is as follows. Suppose a financially unconstrained firm receives a positive productivity shock at time $t - 1$, so that $\ln E_{t-1}(\theta_t)$ is high. This firm increases l_t up to the point that the marginal return on variable capital is equal to its user cost. Alternatively, a financially constrained firm can only invest in variable capital if it has financial wealth available. For this firm $\ln l_t$ is less sensitive to the productivity shock $\ln E_{t-1}(\theta_t)$ and is positively affected by the amount of financial wealth $\ln w_{t-1}^F$. It is important to note that the irreversibility of fixed capital amplifies the effect of financing frictions on variable capital, which implies that variable investment may be significantly financially constrained even after a negative shock, when θ_{t-1} and $E_{t-1}(\theta_t)$ are low. The negative shock implies that k_{t-1} is relatively high, and the firm would prefer to reduce it, but k_t is constrained to be not smaller than $(1 - \delta)k_{t-1}$. In this situation a financially unconstrained firm would choose a relatively high level of l_t , because the two factors of productions are complementary. In contrast, a financially constrained firm is forced to cut variable capital when it does not have enough financial wealth available, and therefore the lower $\ln w_{t-1}^F$ is, the lower $\ln l_t$ is.

This financing constraints test has several useful properties. First, it does not require the estimation of marginal q , but only of the productivity shock θ . Unlike q , θ is not a forward looking variable. Therefore, any error in measuring the profitability of the firm probably implies a smaller measurement error in θ than in q . Moreover, since θ can be estimated from balance sheet data, this test can be easily applied to data sets of small privately owned firms not quoted on the stock market. Second, although it is based on a simple reduced-form investment equation, this test allows for the recovery of the structural parameters α and β . The estimates of α and β provide an additional robustness check of the validity of the model. Third, simulation results presented in the next section show that Eq. (26) is also able to detect the intensity of financing constraints when fixed capital is subject to convex adjustment costs rather than to the irreversibility constraint. The intuition is that in both cases Eq. (26) is well specified, because the information concerning the adjustment costs of fixed capital is summarized by k_t .

3.1. Alternative testing strategy

As an alternative to Eq. (26), one could transform Eq. (15) as follows:

$$\beta \frac{y_t}{l_t} = R[1 + E_{t-1}(\Psi_t^l)] + \varepsilon_t^y, \quad (28)$$

where $\varepsilon_t^y \equiv \beta \frac{E_{t-1}(y_t) - y_t}{l_t}$ is an expectational error. By taking logs and rearranging, we obtain

$$\log l_t = \log \beta + \log y_t - \log\{R[1 + E_{t-1}(\Psi_t^l)] + \varepsilon_t^y\}. \quad (29)$$

Therefore, ε_t^y enters nonlinearly in Eq. (29). If ε_t^y is small relative to $E_{t-1}(\Psi_t^l)$, one can approximate $\log\{R[1 + E_{t-1}(\Psi_t^l)] + \varepsilon_t^y\}$ with $\log\{R[1 + E_{t-1}(\Psi_t^l)]\} + \varepsilon_t^y$, and obtain the

following:

$$\ln l_t = \pi_0 + \pi_1 \ln y_t + \pi_2 \ln w_{t-1}^F + \varepsilon_t^y. \quad (30)$$

In theory, Eq. (30) could be used for the purpose of estimating the intensity of financing constraints. However, our simulations of the calibrated model indicate that ε_t^y is likely to be large because its volatility is driven by the volatility of the idiosyncratic productivity shock. The simulation results also show that the nonlinearity of ε_t^y in Eq. (29) may considerably reduce the precision of the financing constraints test based on Eq. (30), especially when the number of observations in the sample is small. Therefore, in the empirical section of this paper we focus on the estimation of Eq. (26).

4. Simulation results

In this section we use the solution of the model to simulate the activity of many firms that are ex ante identical and subject to an idiosyncratic productivity shock that is uncorrelated across firms and autocorrelated for each firm. We simulate several industries in order to verify whether Eq. (26) is able to detect the intensity of financing constraints on firm investment. We adopt the same methodology commonly used in empirical applications since the seminal paper of Fazzari, Hubbard, and Petersen (1988). Using a priori information to select a subsample of firms more likely to face financing imperfections, we compare the sensitivity of investment to internal finance for this group with respect to the other firms. All simulations assume that prices and the interest rate are constant. As our objective is to analyze the effects of financing constraints at firm level, the partial equilibrium nature of this exercise does not restrict the analysis in any important way. In one set of simulated industries, firms become financially constrained when the borrowing constraint (5) is binding and their internal finance is not sufficient to finance all profitable investment opportunities. In another set of industries, firms are not financially constrained because v is so high that the borrowing constraint (5) is never binding with equality. We also make a further distinction. In one set of industries fixed capital is irreversible and in another, fixed capital is subject to the following quadratic adjustment costs function:

$$\mu(i_t) = b \frac{1}{2} \left[\frac{i_t}{k_{t-1}} \right]^2 k_{t-1}. \quad (31)$$

In the context of our model, Eq. (31) determines the following reduced-form investment equation:

$$\begin{aligned} \frac{i_t}{k_{t-1}} &= -\frac{1}{b} + \frac{q_{t-1}}{b(1 + \phi_{t-1})} + \Phi_{t-1}, \\ \Phi_{t-1} &\equiv \frac{v\lambda_{t-1}}{b(1 + \phi_{t-1})}; \quad q_{t-1} \equiv E_{t-1} \left[\frac{dV_t(w_t, \theta_t, k_t)}{dk_t} \right]. \end{aligned} \quad (32)$$

In the absence of financing frictions both Φ_{t-1} and ϕ_{t-1} are equal to zero, and Eq. (32) simplifies to a linear relationship between marginal q and the investment rate:

$$\frac{i_t}{k_{t-1}} = -\frac{1}{b} + \frac{1}{b} q_{t-1}. \quad (33)$$

The idiosyncratic shock is modeled as follows (in the remainder of the paper we include the subscript i to indicate the i th firm):

$$y_t = \theta_{i,t}^I (\theta_{i,t} k_t^\alpha l_t^\beta) \quad \text{with } \alpha + \beta < 1, \quad (34)$$

where $\theta_{i,t}$ is a persistent shock and $\theta_{i,t}^I$ is an identically and independently distributed (i.i.d.) shock that evolve according to:

$$\ln \theta_{i,t} = \rho \ln \theta_{i,t-1} + \varepsilon_{i,t}, \quad (35)$$

$$0 < \rho < 1; \varepsilon_{i,t} \sim iid(0, \sigma_\varepsilon^2) \quad \text{for all } i, t, \quad (36)$$

$$\ln \theta_{i,t}^I = \varepsilon_{i,t}^I, \quad (37)$$

$$\varepsilon_{i,t}^I \sim iid(0, \sigma_{\varepsilon^I}^2) \quad \text{for all } i, t. \quad (38)$$

The persistent shock θ is necessary to match the volatility and persistence in firm investment. The i.i.d. shock θ^I matches the volatility of profits and ensures that they are negative for a significant share of firms in the simulated industry. Both shocks are important because they allow the simulated firms to observe realistic dynamics for both investment and financial wealth. If we only allow for the persistent shock θ (by setting $\sigma_{\varepsilon^I}^2 = 0$), not only is the volatility of profits of simulated firms too low, but firms also never have negative profits, which are observed for a large share of firm-year observations in the sample used for the empirical analysis in the next section.

The dynamic investment problem is solved using a numerical method (see Appendix C for details). The model is parameterized assuming that the time period is one year. [Table 1](#)

Table 1
Calibrated parameters and matched moments for the simulated industries

The abbreviation ‘‘Q.a.c.’’ refers to the simulated industry with quadratic adjustment costs of fixed capital, whereas ‘‘Irr.’’ refers to the simulated industry with irreversibility of fixed capital. r is the real interest rate; α is the output elasticity of fixed capital; β is the output elasticity of variable capital; δ and δ_I are the depreciation rates of fixed capital and variable capital, respectively; b is the quadratic adjustment costs coefficient; ρ is the autocorrelation coefficient and σ_ε is the standard deviation of the persistent idiosyncratic shock θ ; σ_{ε^I} is the standard deviation of the i.i.d. shock θ^I ; τ is the fraction of the value of fixed capital that can be used as collateral; γ is the exit rate of firms; I/K is the gross fixed investment rate; and CF/K is the ratio of cash flow over fixed capital.

	Parameter values		Empirical restriction	Matched moments		
	Q.a.c.	Irr.		Data	Q.a.c.	Irr.
r	0.02	0.02	Real interest rate	0.02	0.02	0.02
α	0.105	0.08	Returns to scale	0.97	0.97	0.97
β	0.865	0.89	Fixed capital/variable capital	0.5–0.7	0.82	0.68
δ	0.12	0.12	Depr. of fixed capital	0.12	0.12	0.12
δ_I	1	1	Depr. of variable capital	1	1	1
b	0.0002	n.a.	Average (I/K)	0.145	0.176	0.161
ρ	0.8	0.95	Std. (I/K)	0.139	0.126	0.172
σ_ε	0.02	0.01	Autocorr. (I/K)	0.239	0.222	0.200
σ_{ε^I}	0.07	0.04	Std. (CF/K)	0.21	0.14	0.12
τ	$1 - \delta$	$1 - \delta$	Debt/assets ratio	0.20	0.15	0.16
γ	0.94	0.94	6% firms exit each year	6%	6%	6%

summarizes the choice of parameters. The risk-free real interest rate r is equal to 2%, which is the average real return on a one-year U.S. T-bill between 1986 and 2005. The sum of α and β matches returns-to-scale equal to 0.97. This value is consistent with studies on disaggregated data that find returns-to-scale to be just below one (Burnside, 1996). Moreover, because in the model there are no fixed costs of production, even such a small deviation from constant returns is sufficient to generate, for the set of benchmark parameters, average profits in the simulated firms that are relatively large and consistent with the empirical evidence. The parameter β is set to match the ratio of fixed capital to variable capital. In the model, variable capital fully depreciates in one period, and thus we consider as variable capital the sum of material costs and wages, and as fixed capital land, buildings, plant, and equipment. Using yearly data on manufacturing plants from the NBER-CES database (which includes information about the cost of materials), we calculate a fixed capital to variable capital ratio between 0.5 and 0.7 for the 1980 to 1996 period. The other parameters are as follows: the depreciation rate of fixed capital δ is set to 0.12; b , ρ , and σ_ε match the average, standard deviation, and autocorrelation of the fixed investment rate of the U.S. Compustat database as reported in Gomes (2001); $\sigma_{g,t}^2$ matches the standard deviation of the cash flow to fixed capital ratio; ν is set to match the average debt to assets ratio of U.S. corporations; and γ is equal to 0.94, implying that in each period a firm exits with 6% probability, which is consistent with the empirical evidence about firm turnover in the U.S. (source: Statistics of U.S. Businesses, U.S. Census Bureau). The second part of Table 1 reports the matched moments. The simulated industries do not match the empirical moments perfectly, given the presence of nonlinearities in the mapping from the parameters to the moments, but they are sufficiently close for our purposes.

We simulate 50,000 firm-year observations, which can be interpreted as an industry in which we observe every firm in every period of activity, and in which a firm that terminates its activity is replaced by a newborn firm. The initial wealth of a newborn firm is equal to 40% of the average fixed and variable capital of a financially unconstrained firm. This initial endowment ensures that financing constraints are binding for a nonnegligible fraction of firms in the simulated industries. The initial fixed capital of a newborn firm is ex ante optimal, conditional on its initial wealth and the expectation as regards the initial productivity shock. Tables 2–5, report the estimation results from the simulated data. In these tables we do not report the standard deviations of the estimated coefficients, because all coefficients are strongly significant. Panel A in Table 2 reports the estimation results of Eq. (26). It shows that the new test is always able to identify more financially constrained firms because the coefficient of $\ln w_{i,t-1}^F$ is positive in the industries with financing frictions and is equal to zero otherwise. In Panel B in Table 2 we compare the groups of constrained firms to their complementary sample (the test statistic of the difference in the coefficients across groups is not reported because it is always significantly different from zero). We classify firms as financially constrained or not using the average value of the Lagrangian multiplier $\lambda_{i,t}$:

$$\bar{\lambda}_i = \sum_{t=1}^{T_i} \lambda_{i,t}, \quad (39)$$

where T_i is the number of years of operation of firm i . In the industries with financing frictions, the financing constraint is not always binding. This is because firms accumulate wealth and become progressively less likely to face a binding financing constraint. Therefore, the higher $\bar{\lambda}_i$ is, the higher the intensity of financing problems for firm i .

Table 2

The variable capital model with financial wealth; no measurement errors

Two-Stage Least Squares estimates on 50,000 simulated firm-year observations. $l_{i,t}$ is variable capital for simulated firm i in year t ; $k_{i,t}$ is fixed capital; $E_{i,t-1}(\theta_{i,t})$ is expected productivity; $w_{i,t-1}^F$ is financial wealth; $CF_{i,t-1}$ is cash flow, defined as revenues net of interest payments: $CF_{i,t-1} = y_{i,t-1} - \frac{1}{R}b_{i,t}$, where $y_{i,t-1}$ is revenues and $b_{i,t}$ borrowing; and $w_{i,t}^{MAX}$ is the level of financial wealth such that a firm does not expect to be financially constrained in the future. $\ln k_{i,t}$ is instrumented by $\ln k_{i,t-1}$. All the estimated coefficients reported in panel A are statistically significant. In Panels B and C firms are selected in groups according to $\bar{\lambda}$, which is the average value for each firm of the shadow cost of a binding collateral constraint. All the differences across coefficients are statistically significant.

	Quadratic adjustment costs		Irreversibility	
<i>Panel A: Estimation of $\ln l_{i,t} = \pi_0 + \pi_1 \ln E_{i,t-1}(\theta_{i,t}) + \pi_2 \ln k_{i,t} + \pi_3 \ln w_{i,t-1}^F + \varepsilon_{i,t}$</i>				
Industry without financing frictions				
<i>constant</i>		-1.22		-1.262
$\ln E_{i,t-1}(\theta_{i,t})$		7.404		9.170
$\ln k_{i,t}$		0.778		0.726
$\ln w_{i,t-1}^F$		0.000		0.000
R^2		1		1
$\text{corr}(\ln w_{i,t-1}^F, \ln E_{i,t-1}(\theta_{i,t}))$		0.01		-0.48
$\text{corr}(\ln CF_{i,t-1}, \ln E_{i,t-1}(\theta_{i,t}))$		0.52		0.91
Industry with financing frictions				
<i>constant</i>		-0.986		-0.294
$\ln E_{i,t-1}(\theta_{i,t})$		6.256		2.357
$\ln k_{i,t+1}$		0.802		0.903
$\ln w_{i,t-1}^F$		0.006		0.053
R^2		1		0.98
$\text{corr}(\ln w_{i,t-1}^F, \ln E_{i,t-1}(\theta_{i,t}))$		0.03		-0.05
$\text{corr}(\ln CF_{i,t-1}, \ln E_{i,t-1}(\theta_{i,t}))$		0.30		0.79
	Constrained	Complementary sample	Constrained	Complementary sample
<i>Panel B: Coefficient on $\ln w_{i,t-1}^F$ for groups of constrained firms and the complementary sample</i>				
80% most constrained firms	0.008	0.002	0.055	0.014
	($\bar{\lambda} = 0.5\%$)	($\bar{\lambda} = 0.09\%$)	($\bar{\lambda} = 1.9\%$)	($\bar{\lambda} = 0.1\%$)
60% most constrained firms	0.011	0.003	0.066	0.027
	($\bar{\lambda} = 0.6\%$)	($\bar{\lambda} = 0.1\%$)	($\bar{\lambda} = 2.4\%$)	($\bar{\lambda} = 0.3\%$)
40% most constrained firms	0.019	0.004	0.087	0.038
	($\bar{\lambda} = 0.7\%$)	($\bar{\lambda} = 0.2\%$)	($\bar{\lambda} = 3.1\%$)	($\bar{\lambda} = 0.6\%$)
20% most constrained firms	0.037	0.005	0.172	0.046
	($\bar{\lambda} = 1.0\%$)	($\bar{\lambda} = 0.3\%$)	($\bar{\lambda} = 4.2\%$)	($\bar{\lambda} = 0.9\%$)
<i>Panel C: Coefficient on $\ln w_{i,t-1}^F$ for groups of constrained firms and the complementary sample. $\ln w_{i,t-1}^{\max}$ is included among the explanatory variables</i>				
80% most constrained firms	0.007	0.002	0.055	0.012
60% most constrained firms	0.011	0.003	0.063	0.027
40% most constrained firms	0.017	0.004	0.082	0.037
20% most constrained firms	0.043	0.005	0.161	0.045

Table 3

The variable capital model with financial wealth, with and without measurement errors in the productivity shock. Two-Stage Least Squares estimates on 50,000 simulated firm-year observations. $\ln E_{t-1}(\theta_{i,t})^*$ is equal to expected productivity $\ln E_{t-1}(\theta_{i,t})$ plus a measurement error $\kappa_{i,t-1}$, which is independently and identically distributed with standard deviation equal to σ_κ ; $\sigma_{\theta_t^e}$ is the standard deviation of $\ln E_{t-1}(\theta_{i,t})$; $l_{i,t}$ is variable capital for simulated firm i in year t ; $k_{i,t}$ is fixed capital; and $w_{i,t-1}^F$ is financial wealth. $\ln k_{i,t}$ is instrumented by $\ln k_{i,t-1}$. All the estimated coefficients reported in Panel A are statistically significant. Panel B compares the estimates of π_3 for different groups of firms selected according to the average intensity of financing constraints. All the differences across coefficients are statistically significant. When the coefficient π_3 is increasing in the intensity of financing constraints, and is positive for firms with financing frictions, it is reported in italics.

	Quadratic adjustment costs			Irreversibility		
	$\frac{\sigma_\kappa}{\sigma_{\theta_t^e}} = 0$	$\frac{\sigma_\kappa}{\sigma_{\theta_t^e}} = 0.25$	$\frac{\sigma_\kappa}{\sigma_{\theta_t^e}} = 1$	$\frac{\sigma_\kappa}{\sigma_{\theta_t^e}} = 0$	$\frac{\sigma_\kappa}{\sigma_{\theta_t^e}} = 0.25$	$\frac{\sigma_\kappa}{\sigma_{\theta_t^e}} = 1$
<i>Panel A: Estimation of $\ln l_{i,t} = \pi_0 + \pi_1 \ln E_{t-1}(\theta_{i,t})^* + \pi_2 \ln k_{i,t} + \pi_3 \ln w_{i,t-1}^F + \varepsilon_{i,t}$</i>						
Industry without financing frictions						
<i>constant</i>	-1.22	-1.03	-0.451	-1.262	-0.38	0.28
$\ln E_{i,t-1}(\theta_{i,t})^*$	7.404	6.064	1.620	9.170	4.241	0.464
$\ln k_{i,t}$	0.778	0.826	0.993	0.726	0.876	0.992
$\ln w_{i,t-1}^F$	0.000	0.000	-0.001	0.000	0.000	0.000
R^2	1	0.98	0.93	1	0.99	0.99
Industry with financing frictions						
<i>constant</i>	-0.986	-0.714	0.501	-0.294	-0.116	0.143
$\ln E_{i,t-1}(\theta_{i,t})^*$	6.256	5.450	1.844	2.357	1.500	0.232
$\ln k_{i,t}$	0.802	0.810	0.851	0.903	0.935	0.982
$\ln w_{i,t-1}^F$	<i>0.006</i>	<i>0.002</i>	-0.015	<i>0.053</i>	<i>0.046</i>	<i>0.035</i>
R^2	1	0.99	0.96	0.98	0.98	0.98
<i>Panel B: Coefficient of $\ln w_{i,t-1}^F$ for groups of constrained firms and the complementary sample</i>						
80% most constrained firms	<i>0.008</i>	<i>0.004</i>	-0.013	<i>0.055</i>	<i>0.048</i>	<i>0.037</i>
Complementary sample	<i>0.002</i>	-0.001	-0.016	<i>0.014</i>	<i>0.008</i>	<i>0.005</i>
60% most constrained firms	<i>0.011</i>	<i>0.009</i>	-0.008	<i>0.066</i>	<i>0.059</i>	<i>0.048</i>
Complementary sample	<i>0.003</i>	-0.001	-0.016	<i>0.027</i>	<i>0.016</i>	<i>0.009</i>
40% most constrained firms	<i>0.019</i>	<i>0.017</i>	<i>0.003</i>	<i>0.087</i>	<i>0.080</i>	<i>0.067</i>
Complementary sample	<i>0.004</i>	<i>0.000</i>	-0.016	<i>0.038</i>	<i>0.026</i>	<i>0.016</i>
20% most constrained firms	<i>0.037</i>	<i>0.039</i>	<i>0.040</i>	<i>0.172</i>	<i>0.165</i>	<i>0.151</i>
Complementary sample	<i>0.005</i>	<i>0.001</i>	-0.015	<i>0.046</i>	<i>0.035</i>	<i>0.024</i>

Panel B in Table 2 shows that the coefficient of $\ln w_{i,t-1}^F$ also identifies the intensity of financing constraints because its magnitude increases with the magnitude of $\bar{\lambda}_i$ in each industry. Intuitively, the higher the value of $\bar{\lambda}_i$, the more firm i faces a binding financing constraint and the more variable capital is sensitive to financial wealth.

Table 2 also shows that the intensity of financing constraints, and hence the sensitivity of variable capital to financial wealth, is on average larger in industries that face the irreversibility constraint than in those that have convex adjustment costs. The term $\bar{\lambda}$ is higher in the former case because the irreversibility of fixed capital significantly increases the impact of financing frictions on variable capital investment. This happens not only because variable capital is the only factor of production that absorbs wealth fluctuations

Table 4

The variable capital model with financial wealth; different collateral values of capital, industries with irreversibility of fixed capital and with no measurement errors

Two-Stage Least Squares on 50,000 simulated firm-year observations. In the first column the fraction of residual value of fixed capital that is collateralizable is 100%. In the second and third column is 85% and 70%, respectively. $l_{i,t}$ is variable capital for simulated firm i in year t ; $k_{i,t}$ is fixed capital; $E_{i,t-1}(\theta_t)$ is expected productivity; $w_{i,t-1}^F$ is financial wealth; $b_{i,t}$ is debt; and δ is the depreciation rate of fixed capital. $\ln k_t$ is instrumented by $\ln k_{t-1}$. All the estimated coefficients reported in panel A are statistically significant. In Panel B firms are selected in groups according to $\bar{\lambda}$, which is the average value for each firm of the shadow cost of a binding collateral constraint. All the differences across coefficients are statistically significant.

	$b_{i,t} \leq (1 - \delta)k_{i,t}$	$b_{i,t} \leq 0.85(1 - \delta)k_{i,t}$	$b_{i,t} \leq 0.7(1 - \delta)k_{i,t}$			
<i>Panel A: Estimation of $\ln l_{i,t} = \pi_0 + \pi_1 \ln E_{i,t-1}(\theta_{i,t}) + \pi_2 \ln k_{i,t} + \pi_3 \ln w_{i,t-1}^F + \varepsilon_{i,t}$</i>						
constant	-0.294	-0.304	-0.442			
$\ln E_{i,t-1}(\theta_{i,t})$	2.357	2.112	2.434			
$\ln k_{i,t}$	0.903	0.919	0.903			
$\ln w_{i,t-1}^F$	0.053	0.053	0.067			
R^2	0.98	0.98	0.98			
<i>Panel B: Coefficient of $\ln w_{i,t-1}^F$ for groups of constrained firms and the complementary sample</i>						
	Constr.	Compl. sample	Constr.	Compl. sample	Constr.	Compl. sample
80% most constr. firms	0.055	0.014	0.057	0.015	0.073	0.002
	($\bar{\lambda} = 1.9\%$)	($\bar{\lambda} = 0.1\%$)	($\bar{\lambda} = 2.1\%$)	($\bar{\lambda} = 0.1\%$)	($\bar{\lambda} = 2.5\%$)	($\bar{\lambda} = 0.3\%$)
60% most constr. firms	0.066	0.027	0.071	0.028	0.092	0.038
	($\bar{\lambda} = 2.4\%$)	($\bar{\lambda} = 0.3\%$)	($\bar{\lambda} = 2.6\%$)	($\bar{\lambda} = 0.4\%$)	($\bar{\lambda} = 3.0\%$)	($\bar{\lambda} = 0.6\%$)
40% most constr. firms	0.087	0.038	0.094	0.038	0.138	0.047
	($\bar{\lambda} = 3.1\%$)	($\bar{\lambda} = 0.6\%$)	($\bar{\lambda} = 3.3\%$)	($\bar{\lambda} = 0.7\%$)	($\bar{\lambda} = 3.8\%$)	($\bar{\lambda} = 0.9\%$)
20% most constr. firms	0.172	0.046	0.181	0.045	0.252	0.056
	($\bar{\lambda} = 4.2\%$)	($\bar{\lambda} = 0.9\%$)	($\bar{\lambda} = 4.5\%$)	($\bar{\lambda} = 1.0\%$)	($\bar{\lambda} = 5.1\%$)	($\bar{\lambda} = 1.3\%$)

when the irreversibility constraint is binding, but also because when both constraints are binding a firm has too much fixed capital and not enough funds to invest in variable capital. The unbalanced use of the two factors of production reduces revenues and financial wealth and increases the intensity of financing constraints. In contrast, in industries with quadratic adjustment costs, fixed investment is allowed to be negative and a firm can absorb a negative productivity shock by reducing both fixed and variable capital. The other estimated coefficients are consistent with the predictions of the model. In industries without financing frictions, the estimated coefficients π_1 and π_2 are equal to $\frac{1}{1-\beta}$ and $\frac{\alpha}{1-\beta}$. In industries with financing frictions, π_1 and π_2 are also nonlinear functions of the parameters ζ and ω .

The approximations in Eqs. (24) and (25) imply that Eq. (26) is correctly specified also in the presence of financing frictions. It is therefore important to verify that these approximations are correct, and that they do not bias the estimated coefficient of $\ln w_{i,t-1}^F$. First, we verify that the approximation (24) is confirmed by the data. We show this by regressing $\log[1 + E_t(\Psi_{i,t+1}^I)]$ on $\log(w_{i,t}^{\max}/w_{i,t}^F)$. The estimation yields $\eta = 0.024$, with a very high goodness of fit ($R^2 = 0.977$). This relation is depicted in Fig. 1. Second, we

Table 5

The q -model with financial wealth with and without measurement errors in q

Ordinary Least Squares estimates on 50,000 simulated firm-year observations. $i_{i,t}$ is gross fixed investment for simulated firm i in year t ; $k_{i,t-1}$ is fixed capital; $q_{i,t}^*$ is equal to Tobin's marginal $q_{i,t}$ plus a measurement error $\kappa_{i,t-1}^q$, which is independently and identically distributed with standard deviation equal to $\sigma_{\kappa q}$; σ_q is the standard deviation of $q_{i,t}$; $w_{i,t-1}^F$ is financial wealth; $Q_{i,t} = V_{i,t}/w_{i,t}$ is Tobin's average Q , where $V_{i,t}$ is the net present value of expected profits and $w_{i,t}$ is net worth; and $CF_{i,t-1}$ is cash-flow, defined as revenues net of interest payments: $CF_{i,t-1} = y_{i,t-1} - \frac{1}{R}r b_{i,t}$, where $y_{i,t-1}$ is revenues and $b_{i,t}$ borrowing. All the estimated coefficients reported in panel A are statistically significant. In Panels B and C firms are selected in groups according to the average intensity of financing constraints. All the differences across coefficients are statistically significant. When the coefficient α_2 is increasing in the intensity of financing constraints, and is positive for firms with financing frictions, it is reported in italics.

	Quadratic adjustment costs			Irreversibility		
	$\frac{\sigma_{\kappa q}}{\sigma q} = 0$	$\frac{\sigma_{\kappa q}}{\sigma q} = .25$	$\frac{\sigma_{\kappa q}}{\sigma q} = 1$	$\frac{\sigma_{\kappa q}}{\sigma q} = 0$	$\frac{\sigma_{\kappa q}}{\sigma q} = .25$	$\frac{\sigma_{\kappa q}}{\sigma q} = 1$
<i>Panel A: Estimation of $\frac{i_{i,t}}{k_{i,t-1}} = \alpha_0 + \alpha_1 q_{i,t-1}^* + \alpha_2 \frac{w_{i,t-1}^F}{k_{i,t-1}} + \varepsilon_{i,t}$</i>						
Industry without financing frictions						
<i>constant</i>	-9.97	-9.36	-4.81	-3.76	-3.55	-1.83
$q_{i,t-1}$	9.97	9.36	4.88	3.98	3.76	2.01
$\frac{w_{i,t-1}^F}{k_{i,t-1}}$	-0.001	0.000	0.005	0.009	0.010	0.015
R^2	1	0.94	0.49	0.12	0.11	0.07
Industry with financing frictions						
<i>constant</i>	-0.95	-0.86	-0.29	-1.09	-1.01	-0.41
$q_{i,t-1}$	1.01	0.928	0.423	1.20	1.12	0.55
$\frac{w_{i,t-1}^F}{k_{i,t-1}}$	0.020	0.018	0.007	0.006	0.006	0.002
R^2	0.28	0.26	0.12	0.15	0.14	0.07
<i>Panel B: Coefficient on $\frac{w_{i,t-1}^F}{k_{i,t-1}}$ for groups of constrained firms and the complementary sample</i>						
80% most constr. firms	0.028	0.026	0.015	0.005	0.005	0.002
complementary sample	0.013	0.012	0.003	0.015	0.012	0.005
60% most constr. firms	0.041	0.041	0.035	0.007	0.006	0.001
complementary sample	0.015	0.013	0.003	0.008	0.007	0.003
40% most constr. firms	0.072	0.074	0.084	0.008	0.007	0.000
complementary sample	0.016	0.014	0.004	0.008	0.007	0.003
20% most constr. firms	0.122	0.132	0.168	0.042	0.041	0.027
complementary sample	0.017	0.016	0.005	0.008	0.007	0.003
<i>Panel C: Coefficient on $\frac{CF_{i,t-1}}{k_{i,t-1}}$ in the Q-model $\left(\frac{i_{i,t}}{k_{i,t-1}} = \alpha_0 + \alpha_1 Q_{i,t-1} + \alpha_2 \frac{CF_{i,t-1}}{k_{i,t-1}} + \varepsilon_{i,t} \right)$</i>						
80% most constr. firms	0.556	0.562	0.589	0.592	0.611	0.704
complementary sample	0.533	0.537	0.556	0.625	0.630	0.655
60% most constr. firms	0.565	0.571	0.595	0.570	0.596	0.706
complementary sample	0.531	0.535	0.557	0.635	0.641	0.676
40% most constr. firms	0.569	0.575	0.595	0.551	0.582	0.707
complementary sample	0.540	0.544	0.568	0.627	0.637	0.682
20% most constr. firms	0.578	0.582	0.595	0.546	0.581	0.705
complementary sample	0.545	0.550	0.574	0.613	0.626	0.684

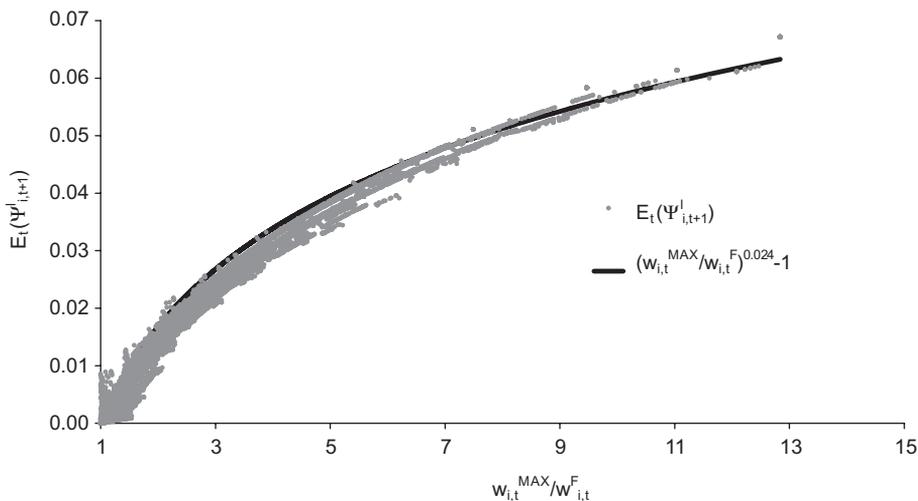


Fig. 1. The Fig. illustrates the relationship between financial wealth and financing constraints in a simulated industry with financing imperfections and irreversibility of fixed capital. The term $E_t(\Psi_{i,t+1}^l)$ measures the intensity of financing constraints, $w_{i,t}^F$ is financial wealth, and $w_{i,t}^{max}$ is the level of financial wealth such that a firm does not expect to be financially constrained now or in the future.

take the logs of Eq. (25) and we estimate it with OLS. The R^2 of the regression is 0.91, suggesting that the effect of the omitted variable $\ln w_{t-1}^{max}$ in Eq. (26) should be absorbed by $\ln E_{t-1}(\theta_t)$ and $\ln k_t$, and should not bias the coefficient of $\ln w_{t-1}^F$ significantly. We verify this claim by estimating a version of Eq. (26) in which $\ln w_{i,t-1}^{max}$ is explicitly included as a regressor. Panel C in Table 2 reports the estimation results, which are very similar to those illustrated above, and confirm that the coefficient of $\ln w_{i,t-1}^F$ is a reliable indicator of the intensity of financing constraints.

So far we have estimated the variable capital equation under the assumption that all variables are perfectly observable. However, in reality the productivity shock $\theta_{i,t}$ is estimated using balance sheet data. Table 3 reports the estimation results of Eq. (26), where $\ln E_{t-1}(\theta_{i,t})$ is observed with noise, that is,

$$\ln E_{t-1}(\theta_{i,t})^* = \ln E_{t-1}(\theta_{i,t}) + \kappa_{i,t-1},$$

where $\kappa_{i,t-1}$ is an i.i.d. error drawn from a normal distribution with mean zero and variance σ_κ^2 . The first column of Table 3 replicates the results in the first column of Table 2. The second and third columns include a measurement error in $\ln E_{t-1}(\theta_{i,t})$, with a “noise-to-signal” ratio (the ratio of σ_κ^2 to the variance of $\ln E_{t-1}(\theta_{i,t})$) equal to 0.25 and one, respectively. The next three columns repeat the same analysis for the economy with the irreversibility constraint. The results show that measurement errors cause a negative bias in the coefficient of $\ln w_{i,t-1}^F$. But because the bias is small, this coefficient is still a reliable indicator of the intensity of financing constraints. It is positive only for financially constrained firms, and a higher value of this coefficient for a group of firms always signals that this group is more financially constrained than the complementary sample. The only exception is in the third column: in this case in which the measurement error is very large and firms are not very constrained (in the economy with quadratic adjustment costs, $\bar{\lambda}$ is

much smaller than 1% for all firms except the most constrained quintile of firms), then the coefficient on $\ln w_{i,t-1}^F$ becomes negative, even though it is still increasing in the intensity of financing constraints.

The measurement error in the productivity shock has little effect on the coefficient of $\ln w_{i,t-1}^F$, because these two variables are nearly uncorrelated in the industries with financing constraints (see Table 2). This happens despite lagged cash flow, which is one of the determinants of financial wealth, being positively correlated to the productivity shock. There are two reasons for the low correlation between $\ln E_{t-1}(\theta_{i,t})$ and $\ln w_{i,t-1}^F$. First, firms that face financing imperfections accumulate financial wealth. This means that $w_{i,t-1}^F$ increases as the accumulated savings increase, and it becomes less sensitive to current fluctuations in cash flow. Second, Eq. (7) shows that the net worth of the firm is the sum of financial wealth w_t^F and the residual value of fixed capital $(1 - \delta)k_t$. Because the productivity shock is persistent, when $\theta_{i,t-1}$ and $E_{t-1}(\theta_{i,t})$ are low, it is also likely that $\theta_{i,t-2}$ was low, so that the firm did not invest in fixed capital in the past, and a larger fraction of its wealth $w_{i,t-1}$ was invested in financial wealth $w_{i,t-1}^F$. The same reasoning applies when $E_{t-1}(\theta_{i,t})$ is high. This “composition effect” implies a negative correlation between financial wealth and the productivity shock, and it counterbalances the positive correlation effect induced by the cash flow.

We have so far assumed that the residual value of capital is entirely collateralizable. In other words, there is no discount in the liquidation value of the firm’s fixed assets. This assumption increases the leverage of the simulated firms and gets it closer to the empirical value. In reality, however, distressed firms often sell capital at fire-sale prices. Therefore, in Table 4 we estimate Eq. (26) for industries with different values of v . The first column replicates the results of Table 2, with $v = 1 - \delta$. The second and third columns consider $v = 0.85(1 - \delta)$, and $v = 0.7(1 - \delta)$, respectively. They show that the lower the collateral value of capital, the higher the intensity of financing constraints and the coefficient of $\ln w_{i,t-1}^F$.

Summing up, the simulation results illustrated in Tables 2–4 suggest that the coefficient on $\ln w_{i,t-1}^F$ in Eq. (26) is a precise and reliable indicator of financing constraints, even in the presence of different types of adjustment costs of fixed capital and large observational errors in the productivity shock.

In the remainder of this section we compare the performance of this new test with a test based on the q model of fixed capital:

$$\frac{i_{i,t}}{k_{i,t-1}} = \alpha_0 + \alpha_1 q_{i,t-1} + \alpha_2 \frac{w_{i,t-1}^F}{k_{i,t-1}} + \varepsilon_{i,t}. \tag{40}$$

Table 5 shows the estimation results of Eq. (40). In the $\frac{\sigma_{\varepsilon q}}{\sigma q} = 0.25$ and $\frac{\sigma_{\varepsilon q}}{\sigma q} = 1$ columns there is a measurement error in q , with a noise-to-signal ratio equal to 0.25 and one, respectively. In the first half of Table 5, adjustment costs are quadratic. In the absence of financing frictions, the investment ratio $\frac{i_{i,t}}{k_{i,t-1}}$ is determined by Eq. (33) and therefore $q_{i,t-1}$ is a sufficient statistic for $\frac{i_{i,t}}{k_{i,t-1}}$. As a consequence, the coefficient on $\frac{w_{i,t-1}^F}{k_{i,t-1}}$ is equal to zero. In the presence of financing frictions the coefficient on $\frac{w_{i,t-1}^F}{k_{i,t-1}}$ is positive, significant, and increasing in the intensity of financing constraints, even in the presence of measurement errors, because $\frac{i_{i,t}}{k_{i,t-1}}$ is determined by Eq. (32), and $\frac{w_{i,t-1}^F}{k_{i,t-1}}$ is negatively correlated with the omitted term $1 + \phi_{i,t-1}$. Therefore, the first half of Table 5 shows that when adjustment

costs are convex, Eq. (40) does a good job of identifying financing constraints, even in the presence of measurement errors in q . In contrast, in the second part of Table 5 we consider industries that face irreversibility of fixed capital. Here q is no longer a sufficient statistic for investment, and the coefficient on $\frac{w_{i,t-1}^F}{k_{i,t-1}}$ is positive for unconstrained firms because financial wealth conveys relevant information about investment. Moreover, the coefficient on $\frac{w_{i,t-1}^F}{k_{i,t-1}}$ is small for financially constrained firms because for those firms, most of the fluctuations in wealth are absorbed by variable capital. As a consequence, fixed capital investment is more sensitive to financial wealth for less constrained than for more constrained firms for almost all of the sorting criteria. Thus, Eq. (40) is not useful for identifying financing constraints, consistently with Gomes (2001), Pratap (2003), Moyen (2004), and Hennessy and Whited (2007).

A more direct comparison with the previous literature is provided in Panel C in Table 5, where we use average Q to replace the unobservable marginal q , and we use the cash flow ratio $\frac{CF_{i,t-1}}{k_{i,t-1}}$ as the explanatory variable that captures financing frictions. The results show that the cash flow coefficient is highly significant both for constrained and unconstrained industries, as also found by Moyen (2004). However, this coefficient is not a good indicator of the presence of financing constraints in the presence of fixed capital irreversibility.

5. Empirical evidence

In this section we verify empirically the validity of the new test of financing constraints described in the previous section on a sample of small and medium Italian manufacturing firms. The sample is obtained by merging two data sets provided by Mediocredito Centrale. The first data set is a balanced panel of more than 5,000 firms with company accounts data for the 1982 to 1991 period.⁷ This is a subset of the broader data set of the Company Accounts Data Service, which is the most reliable source of information on the balance sheet and income statements of Italian firms, and is often used in empirical studies on firm investment (e.g., Guiso and Parigi, 1999). The second data set consists of the four Mediocredito Centrale Surveys (1992, 1995, 1998, and 2001) on small and medium Italian manufacturing firms. Each Survey covers the activity of a sample of more than 4,400 small and medium manufacturing firms in the three previous years. The samples are selected balancing the criterion of randomness with that of continuity. Each survey contains three consecutive years of data. After the third year, two-thirds of the sample is replaced and the new sample is then held constant for the three following years.

The information provided in the surveys includes detailed qualitative information on property structure, employment, R&D and innovation, internationalization, and financial structure. Among the financial information, each Survey asks specific questions about financing constraints. In addition to this qualitative information, Mediocredito Centrale also provides, for most of the firms in the sample, an unbalanced panel with some balance sheet data items going back as far as 1989. Examples of published papers that use the Mediocredito Centrale surveys are Basile, Giunta, and Nugent (2003) and Piga (2002).

⁷The original sample had balance sheet data from 1982 to 1994, but we discarded the last three years of balance sheet data (1992, 1993 and 1994) from the sample, because of discrepancies and discontinuities in some of the balance sheet items, probably due to changes in accounting rules in Italy in 1992.

The main data set used in this section is obtained by merging the firms in the balanced panel of the Centrale dei Bilanci with the firms in the 1992 Mediocredito Survey. The merged sample consists of 812 firms, for which we have a unique combination of very detailed balance sheet data and detailed qualitative information about financing constraints. As a robustness check, in Section 6.3 we consider an alternative data set based on the 1998 and 2001 surveys. This data set is larger but has less detailed balance sheet data and less precise information about financing constraints.

Regarding the main data set, we eliminate firms without detailed information concerning the composition of fixed assets (that do not distinguish between plant and equipment on the one side and land and building on the other side), which leaves us with 561 firms. We further eliminate firms that merged or firms that split during the sample period. The remaining sample comprises 415 firms, virtually none of which is quoted on the stock markets. The information on financing constraints is contained in the investment section of the 1992 Survey. This section requests detailed information regarding the most recent investment projects aimed at improving the firm's production capacity. In particular, the survey asks for both the size of the project and the years in which the project was undertaken. Approximately 95% of all the answers relate to projects undertaken between 1988 and 1991. Among the financial information, the firm is asked whether it had difficulties in financing the indicated project because of: (a) "lack of medium-to-long-term financing;" (b) "high cost of banking debt;" (c) "lack of guarantees."

It is worthwhile to note that the selection of firms in this sample is biased towards less financially constrained firms for at least two reasons: (i) the balanced panel only includes firms that have been continually in operation between 1982 and 1991, thereby excluding new firms and firms that ceased to exist during the same period because of financial difficulties; and (ii) by eliminating mergers we eliminate firms in profitable businesses that merged with other companies because of their financing problems.

For the empirical specification of the financing constraints test we consider the following production function:

$$y_{i,t} = \theta_{i,t} k_{i,t-1}^{\alpha} l_{i,t}^{\beta} n_{i,t}^{\gamma} \quad (41)$$

All variables are in real terms, and are defined as follows: $y_{i,t}$ is total revenues (during period t , firm i); $k_{i,t-1}$ is the replacement value of plant, equipment, and intangible fixed capital (end of period $t-1$, firm i); $l_{i,t}$ is the cost of the use of materials (during period t , firm i); and $n_{i,t}$ is labor cost (during period t , firm i). Detailed information about all the variables is reported in Appendix D. With respect to Eq. (1) in the theoretical model, Eq. (41) includes also labor as factor of production and lags fixed capital by one period. That is, we assume that fixed capital installed in period t will become productive from period $t+1$ on. Under these assumptions the first-order condition for variable capital is still represented by Eq. (15). By using Eq. (41) in (15), we obtain

$$\beta E_t(\theta_{i,t+1}) k_{i,t}^{\alpha} l_{i,t+1}^{\beta-1} n_{i,t+1}^{\gamma} = R[1 + E_t(\Psi_{i,t+1}^l)]. \quad (42)$$

Eq. (42) implies that Proposition 1 still holds, conditional also on $n_{i,t}$. Moreover, we can rearrange Eq. (42) and lag it by one period to obtain the following reduced-form variable capital equation:

$$\ln l_{i,t} = \pi_0 + a_i + d_t + \pi_1 \ln \theta_{i,t-1} + \pi_2 \ln k_{i,t-1} + \pi_3 \ln n_{i,t} + \pi_4 \ln w_{i,t-1}^F + \varepsilon_{i,t}, \quad (43)$$

where $\varepsilon_{i,t}$ is the error term and $\ln \theta_{i,t-1}$ is the productivity shock, which is derived by taking the expectation of Eq. (35) and noting that $\ln E_{t-1}(\theta_{i,t}) = \rho + \frac{\sigma_\theta^2}{2} + \ln \theta_{i,t-1}$. The term $\rho + \frac{\sigma_\theta^2}{2}$ is included in the constant term. The coefficient π_4 measures the intensity of financing constraints. Under the assumption of no financing constraints the reduced-form coefficients π_1, π_2 , and π_3 can be used to recover the structural parameters α, β , and γ :

$$\pi_1 = \frac{1}{1 - \beta}; \quad \pi_2 = \frac{\alpha}{1 - \beta}; \quad \pi_3 = \frac{\gamma}{1 - \beta}. \tag{44}$$

In the model the user cost of variable capital is constant and equal to R for financially unconstrained firms. In reality the user cost of capital may vary across firms and over time for several reasons unrelated to financing imperfections, such as transaction costs, taxes, and risk. Therefore in Eq. (43) we also include firm and year dummy variables, respectively, a_i and d_t . These capture, among other things, the changes in the user cost of capital across firms and over time for all firms.

We estimate the productivity shock $\ln \theta_{i,t-1}$ from the Solow residual of the production function at the beginning of period t . The method used is robust to the presence of decreasing returns to scale and to heterogeneity in technology (see Appendix E for details).

We compute $w_{i,t}^F$, net financial wealth of firm i at the beginning of period t , by using the budget constraint (9) at time $t - 1$ to substitute $b_{i,t}$ in (8):

$$\begin{aligned} w_{i,t}^F &= \Pi_{i,t} + R_t(w_{i,t-1}^F - d_{i,t-1}), \\ \Pi_{i,t} &\equiv y_{i,t} - R_t(l_{i,t} + i_{i,t}). \end{aligned} \tag{45}$$

In the model, $\Pi_{i,t}$ are beginning-of-period t profits generated from the investment in period $t - 1$. We therefore estimate $\Pi_{i,t}$ as the operating profits during period $t - 1$ (value of production minus the cost of production inputs). Moreover, we estimate $(w_{i,t-1}^F - d_{i,t-1})$ as the net short-term financial assets (after dividend payments) plus the stock of finished good inventories at the beginning of period $t - 1$. We include the stock of finished good inventories because most of such goods will be transformed into cash flow during period $t - 1$. The term R_t is equal to one plus the average real interest rate during period $t - 1$.

The concave transformation of wealth in Eq. (24) can be computed only if $w_{i,t}^F$ is positive. The simulations of the model show that, for reasonable parameter values, financial wealth is always positive in an economy with financing frictions. This is because such frictions also reduce the maximum amount of borrowing and give firms an incentive to accumulate financial assets. The empirical data are consistent with this finding, because the variable $w_{i,t}^F$ is positive for 95.2% of the firm-year observations. Among the 4.8% negative observations, nearly half are excluded as outliers. In order to include the remaining negative observations, we consider an alternative definition of financial wealth based on the following modification of the borrowing constraint (9):

$$b_{i,t+1} \leq v k_{i,t+1} + \bar{b}_i, \tag{46}$$

where \bar{b}_i represents the collateral value of firm i in addition to the residual value of its assets and can be interpreted as intangible collateral assets (for example, from relationship lending). In this case it is appropriate to modify Eq. (24) as follows:

$$1 + E_t(\Psi_{i,t+1}^I) = (w_{i,t}^{\max} / \bar{w}_{i,t}^F)^\eta \quad \text{if } \bar{w}_{i,t}^F \leq w_{i,t}^{\max}, \tag{47}$$

$$\bar{w}_{i,t}^F = w_{i,t}^F + \bar{b}_i. \tag{48}$$

We estimate \bar{b}_i as the average borrowing of a firm in excess of the collateral value of the firm's fixed assets. The value \bar{b}_i is found to be positive for 125 firms (30% of the total). The term $\bar{w}_{i,t}^F$ is positive for 97.5% firm-year observations.

The estimation of Eq. (43) is complicated by the endogeneity of the regressors. First, all the regressors are most likely correlated to the firm-specific effect a_i . Second, $\ln n_{i,t}$ is endogenous because it is simultaneously determined with $\ln l_{i,t}$. Third, the other right-hand side variables are predetermined, but they may still be endogenous and correlated to $\varepsilon_{i,t}$. In other words, if all the relevant information about future expected productivity is summarized by $\ln \theta_{i,t-1}$, then $\varepsilon_{i,t}$ should be uncorrelated with the predetermined regressors; otherwise, an unobservable and persistent productivity shock in period $t-1$ may at the same time affect $w_{i,t-1}^F$, $k_{i,t-1}$, and $\varepsilon_{i,t}$ and cause an error-regressor correlation. The same problem may be caused by persistent measurement errors. In this case, a suitable estimation strategy is to first difference Eq. (43) in order to eliminate the unobservable firm-specific effect a_i , and then estimate it with a GMM estimation technique, using the available lagged levels and first differences of the explanatory variables as instruments. In this case, the set of instruments is different for each year and Eq. (43) is estimated as a system of cross-sectional equations, each one corresponding to a different period t (Arellano and Bond, 1991). More recent lags are likely to be better instruments, but they may be correlated with the error term if the unobservable productivity shock is highly persistent. The test of overidentifying restrictions can be used to assess the orthogonality of the instruments with the error term. Moreover, under the assumption that $E(\Delta z_{i,t-j}, a_i) = 0$, with $z_{i,t} = \{\ln \theta_{i,t}, \ln k_{i,t}, \ln w_{i,t}^F, \ln n_{i,t}\}$, $\Delta z_{i,t-j}$ is a valid instrument for Eq. (43) estimated in levels. Blundell and Bond (1998) propose a System GMM estimation technique that uses both the equation in levels (instrumented using lagged first differences), and the equation in first differences (instrumented using lagged levels). Using Monte Carlo simulations, they show that the System GMM estimator is much more efficient than the simple GMM estimator when the regressors are highly persistent, and when the number of observations is small. These properties are particularly useful in our context. Table 6 shows the test of the validity of the instruments for the estimation of Eq. (43). Panel A reports the p -value of the Hansen J statistic that tests the orthogonality of the instruments. Panel B in Table 6 reports the F statistic of the excluded instruments and the partial- R^2 from Shea (1997). The table shows that the $t-1$ to $t-3$ first differences as instruments for the equation in levels and the $t-3$ levels as instruments for the equation in first differences are not rejected by the orthogonality test and are sufficiently correlated with the regressors for the coefficients of Eq. (43) to be identified.

The primary objective of this empirical analysis is to verify that the coefficient of $\ln w_{i,t-1}^F$ in Eq. (43) is a precise indicator of the intensity of financing constraints. We do so by using the qualitative information provided by the Mediocredito Survey, which allows us to identify those firms that are more likely to be financially constrained. We also select firms according to several exogenous criteria commonly used in the previous literature as indicators of financing imperfections. In particular, we include dividend policy. Firms that have higher cost (or rationing) of external finance than internally generated finance are less likely to distribute dividends. Therefore the observed dividend policy should be correlated with the intensity of financing constraints. We also include size and age, as smaller and younger firms usually are more subject to informational asymmetries that may generate financing constraints. More specifically, we estimate Eq. (43) for subsamples of firms selected according to the dummy variable $D_{i,t}^X$, which is equal to one if firm i belongs to the

Table 6

Test of the validity of the instruments for the estimation of the following variable capital investment equation: $\ln I_{i,t} = \pi_0 + a_i + d_t + \pi_1 \ln \theta_{i,t-1} + \pi_2 \ln k_{i,t-1} + \pi_3 \ln n_{i,t} + \pi_4 \ln w_{i,t-1}^F + \varepsilon_{i,t}$; sample of 415 small and medium Italian manufacturing firms, 1986 to 1991 period

In Panel A we estimate the model separately for each sample year, and we report the Hansen test of overidentifying restrictions, which is robust to heteroskedasticity and autocorrelation of unknown form. In the first column we estimate the equation in levels using lagged first differences of both dependent and independent variables as instruments. In the second and third columns we estimate the equation in first differences using lagged levels as instruments. In Panel B we estimate jointly all sample years, and we report the test statistics of the validity of the instruments. $\theta_{i,t-1}$ is the estimated productivity shock for firm i in period $t - 1$; $k_{i,t}$ is the replacement value of the plant, equipment and other intangible fixed assets; $l_{i,t}$ is the usage of materials; $n_{i,t}$ is labor cost; and $w_{i,t-1}^F$ is financial wealth.

	$t - 1$ to $t - 3$ first differences as instruments of the levels equation ^a	$t - 2$ to $t - 3$ levels as instruments of the equation in first differences	$t - 3$ to $t - 4$ levels as instruments of the equation in first differences
<i>Panel A: Hansen J statistic (p-value)—cross-sectional equations</i>			
1986	0.34	0.67	
1987	0.46	0.03	0.87
1988	0.81	0.53	0.89
1989	0.51	0.58	0.66
1990	0.27	0.51	0.67
1991	0.93	0.03	0.11
<i>Panel B: First-stage regressions statistics</i>			
$\ln k_{i,t-1}$			
Shea's partial R^2	0.01	0.13	0.07
F stat. (p -val.)	1 (0.60)	24 (0.000)	10 (0.000)
$\ln n_{i,t}$			
Shea's partial R^2	0.025	0.05	0.03
F stat. (p -val.)	1.1 (0.37)	10 (0.000)	5.4 (0.000)
$\ln w_{i,t-1}^F$			
Shea's partial R^2	0.16	0.16	0.01
F stat. (p -val.)	5.2 (0.000)	20 (0.000)	2.3 (0.013)
$\ln \theta_{i,t-1}$			
Shea's partial R^2	0.44	0.31	0.05
F stat. (p -val.)	60.4 (0.000)	44 (0.000)	12 (0.00)
Number of obs.	1970	1970	1970

^aWe include the $t - 1$ to $t - 3$ first differences of the regressors and the $t - 2$ to $t - 3$ first differences of the dependent variable.

specific group x , and zero otherwise. Among the direct criteria, D^{hs} identifies firms that declare too high a cost of banking debt (13.7% of all firms), and D^{lc} identifies firms with a lack of medium-to-long-term financing (13.2% of all firms).⁸ Among the indirect criteria,

⁸We do not select firms according to the question concerning “lack of guarantees” because only 2% of firms answer positively, and almost all of those are already included in the D^{lc} and D^{hs} groups. Also, among all the firms in the sample, 8% did not declare any investment project in the Mediocredito Survey and hence did not answer the questions about financing constraints. We keep these firms in the unconstrained sample, but one may also argue that perhaps some of these firms did not invest precisely because they may have been financially constrained. In

D^{age} identifies firms founded after 1979 (16% of all firms), D^{divpol} identifies firms with zero dividends in any period (33.4% of all firms), and D^{size} identifies firms with less than 65 employees (in 1992) (16% of all firms).

We estimate the coefficients of Eq. (43) separately for each group of firms and for the complementary sample by interacting the above criteria with the explanatory variables, the constant, the yearly dummies, and all the instruments. The D^{hs} and D^{lc} dummies are potentially endogenous, because an unobservable shock may be simultaneously correlated with the likelihood of declaring financing constraints and with the error term in Eq. (43). However, this problem is not likely to bias the GMM estimates of Eq. (43) because we exclusively adopt cross-sectional selection rules. In other words, Heckman (1979) shows that the selection bias can be represented as an omitted variable problem. That said, we do not allow firms to wander in and out of the constrained group, and therefore the omitted term is also constant over time for each firm and is absorbed by the fixed effect in the estimation. Because the GMM estimator used in the paper is based on first differences, it is robust to this type of cross-sectional bias.

Another potential problem is measurement error in the Survey. For example, at the time of the 1992 Survey, Mediocredito Centrale was a state-controlled financial institution whose main objective was to provide subsidized credit to small and medium firms. Therefore it may be the case that those firms declaring “lack of medium-long term financing” were actually sending strategic messages to the institution. However, virtually all of the subsidized credit administered by Mediocredito Centrale has been directed to the South of Italy. Indeed, among the 415 firms in the 1992 Mediocredito Survey none of the firms from North and Central Italy had benefited from any subsidized credit, while as much as 58.7% of the firms from the South had. Therefore, this problem can be controlled for by excluding from the D^{lc} group firms from South Italy, which represent 5% of the total (dummy D_{-south}^{lc}).

Another problem is that the $D^{lc} = 1$ group may include some distressed firms that need more banking debt in order to survive, not because they need to finance a profitable project. The structure of the 1992 Survey, which only allows firms to declare financing problems if they actually undertook a new investment project, should avoid this problem. Nonetheless, we control for this possibility by considering an alternative selection criterion that also excludes from the D^{lc} group the decile of firms with the lowest average ratio of gross profits over sales (dummy $D_{-s.\&lowyield}^{lc}$).

Table 7 illustrates the summary statistics. The whole sample consists almost entirely of small firms: 50% of the firms have fewer than 123 employees and 90% fewer than 433. Virtually all of these firms are privately owned and not quoted on the stock market. Likely financially constrained firms do not show significant differences with respect to the other firms in terms of size, growth rate of sales, investment rates, riskiness (volatility of output), and gross income margin. The most noticeable differences concern financial structure. Firms that declare financing constraints are less wealthy, on average pay higher interest rates on banking debt, and have a lower net income margin.

Table 8 reports the estimation results of Eq. (43) for the whole sample and for the groups selected according to the “direct criteria” dummies. In the first column we use the

(footnote continued)

order to control for this possibility we repeat the analysis excluding such firms from the sample and obtain results very similar to those reported in the following sections.

Table 7

Mediocredito data set on Italian Manufacturing firms; summary statistics, 1982–1991 period

Standard deviations in parentheses. The “ $D^{hs} = 1$ ” column refers to firms that declare “too high a cost of banking debt”, and the “ $D^{lc} = 1$ ” column refers to firms that declare “lack of medium-to-long-term financing”. K = fixed assets; Financial wealth = operating profits during period $t - 1$ (value of production minus the cost of production inputs) plus the net short-term financial assets (after dividend payments) and the stock of finished goods inventories at the beginning of period $t - 1$ multiplied by one plus the nominal interest rate.

	All firms	$D^{hs} = 1$	$D^{lc} = 1$
Mean fixed assets ^a	6331	3136	4140
Median fixed assets ^a	2442	2200	3064
Mean number of employees	207	141	175
Median number of employees	123	119	131
90th percentile of employees	433	249	364
Short term banking debt/ K	0.50 (0.15)	0.54 (0.13)	0.51 (0.14)
Long term banking debt/ K	0.10 (0.08)	0.11 (0.09)	0.11 (0.07)
Average cost of debt ^b	0.066 (.035)	0.075 (.037)	0.076 (0.036)
Gross income margin	0.066 (0.058)	0.065 (0.046)	0.068 (.063)
Net income margin	0.018 (0.05)	0.01 (0.034)	0.014 (0.05)
Net sales growth	0.11 (0.19)	0.11 (0.17)	0.12 (0.20)
Financial wealth/ K^c	1.50 (1.64)	1.11 (1.02)	1.19 (1.06)
Cash flow/ K	0.41 (0.57)	0.29 (0.26)	0.35 (0.43)
Gross fixed Investment/ K	0.30 (0.34)	0.28 (0.30)	0.30 (0.28)
Volatility of output ^d	1.18 (0.22)	1.17 (0.24)	1.21 (0.25)
Number of firms	415	63	56
Number of observations	4150	630	560

^aValues are in billions of Italian Lire, 1982 prices. 1 billion lire was equal to 0.71 million US\$ at the 1982 exchange rate.

^bInterest paid on banking debt divided by total banking debt.

^cThe largest 1% and smallest 1% of the observations are excluded from the computation of this statistic.

^dAverage of the standard deviation of the growth rate of sales.

data from the 1986 to 1991 period, the period for which the full set of instruments are available. In the other columns we estimate the model for the shorter 1988 to 1991 period.⁹

The full-sample estimates in the first two columns show that the coefficients on $\ln \theta_{i,t-1}$, $\ln k_{i,t-1}$, and $\ln n_{i,t}$ are all significant and have the expected sign and size. The coefficient on $\ln w_{i,t-1}^F$ is very small in magnitude and not significantly different from zero. This suggests that financing constraints do not affect a large share of the firms in the sample, and is consistent with the information from the Mediocredito Survey, where only 22% of the firms state some problem in financing investment.

The remaining columns in Table 8 allow all the coefficients to vary across the subgroups of firms. In the third and fourth columns, the first set of coefficients is relative to the group of firms that declare their cost of debt is too high ($D^{hs} = 0$). The second set of coefficients are relative to all the regressors multiplied by D^{hs} . They represent the difference between the coefficient on the likely constrained firms ($D^{hs} = 1$) and that on the complementary sample ($D^{hs} = 0$). Therefore, the t -statistic of this second set of estimates can be used to test the equality of the coefficients across groups. Column “1” uses the definition of financial

⁹We restrict the sample because the Mediocredito Survey refers to the 1989–1991 period, but 5% of the investment projects surveyed actually started in 1988.

Table 8

Financing constraints test based on the following reduced form variable capital investment equation: $\ln l_{i,t} = \pi_0 + a_i + d_t + \pi_1 \ln \theta_{i,t-1} + \pi_2 \ln k_{i,t-1} + \pi_3 \ln n_{i,t} + \pi_4 \ln w_{i,t-1}^F + \varepsilon_{i,t}$; sample of 415 small and medium Italian manufacturing firms, 1986–1991 period

*Significant at the 10% level; **significant at the 5% level; ***significant at the 1% level. The t -statistic is reported in parentheses. In columns “(1)” financial wealth is defined by Eq. (45). In columns “(2)” financial wealth is defined by Eq. (48). In each “ D^x ” column, a dummy D^x , which identifies firms that are more likely to be financially constrained, is interacted with the constant and all the regressors. D^{hs} is equal to one for firms that declare too high a cost of banking debt, and is equal to zero otherwise; D^{lc} is equal to one for firms with a lack of medium-to-long-term financing, and is equal to zero otherwise; D_{-south}^{lc} is equal to one if both D^{lc} is equal to one and the firm is not from south Italy, and is equal to zero otherwise; $D_{-s.\&lowyield}^{lc}$ is equal to one if both D_{-south}^{lc} is equal to one and the firm is not among the lowest decile of profitability. The coefficients are estimated with a two-step robust System GMM estimator (Blundell and Bond, 1998). The finite-sample correction to the two-step covariance matrix is derived by Windmeijer (2005). $\theta_{i,t-1}$ is the estimated productivity shock for firm i in period $t - 1$; $k_{i,t}$ is the replacement value of the plant, equipment and other intangible fixed assets; $l_{i,t}$ is the usage of materials; $n_{i,t}$ is labor cost; and $w_{i,t-1}^F$ is financial wealth. The smallest 1% and largest 1% of the first differences of the regressors and of the dependent variable are eliminated as outliers. Year dummy variables are entered as strictly exogenous regressors. Instruments for the equation in levels are $t - 1$ to $t - 3$ first differences of the regressors and $t - 2$ to $t - 3$ first differences of the dependent variable. Instruments for the equation in first differences are $t - 3$ levels of the regressors and of the dependent variable. The F -test reports the test of joint significance of all estimated coefficients. The Hansen test of overidentifying restrictions is reported, which is robust to autocorrelation and heteroskedasticity of unknown form. The p -value in the last row is the probability of rejecting H_0 when it is true.

	All firms (86–91)	1988–1991 sample								
		All firms (88–91)	D^{hs}		D^{lc}		D_{-south}^{lc}		$D_{-s.\&lowyield}^{lc}$	
		(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)	
$\ln k_{i,t-1}$	0.30*** (3.6)	0.17* (1.9)	0.08 (0.8)	0.05 (0.5)	0.13 (1.4)	0.18** (2.0)	0.15 (1.5)	0.18** (1.9)	0.16* (1.7)	0.25** (2.3)
$\ln n_{i,t}$	0.62*** (5.1)	0.66*** (4.8)	0.86*** (5.6)	0.99*** (6.3)	0.82*** (5.4)	0.66*** (4.6)	0.81*** (5.2)	0.80*** (4.6)	0.77*** (5.1)	0.74*** (5.3)
$\ln \theta_{i,t-1}$	1.94** (2.0)	1.57* (1.7)	1.56* (1.6)	2.01* (1.8)	1.80** (2.0)	2.27*** (2.6)	1.99** (2.1)	2.43** (2.5)	1.99** (2.1)	2.72*** (2.7)
$\ln w_{i,t-1}^F$	0.01 (0.4)	-0.02 (-0.5)	-0.05 (-1.5)	0.02 (0.4)	-0.03 (-0.7)	0.00 (0.1)	-0.02 (-0.6)	0.03 (0.5)	-0.03 (-0.8)	0.03 (0.7)
$\ln k_{i,t-1} * D_{i,t}$			0.17 (0.7)	0.10 (0.5)	0.29 (1.4)	0.22 (1.0)	0.25 (1.2)	0.22 (1.0)	0.25 (1.0)	0.14 (0.7)
$\ln n_{i,t} * D_{i,t}$			-0.49** (-2.3)	-0.63*** (-2.6)	-0.51*** (-2.8)	-0.26 (-0.9)	-0.46** (-2.5)	-0.43** (-2.1)	-0.48** (-2.2)	-0.37* (-1.8)
$\ln \theta_{i,t-1} * D_{i,t}$			-1.34 (-0.4)	-2.53 (-0.6)	5.23 (0.9)	5.82 (1.1)	4.17 (0.7)	4.52 (0.5)	5.59 (0.8)	4.94 (0.8)
$\ln w_{i,t-1}^F * D_{i,t}$			0.33*** (2.9)	0.24** (1.9)	0.19* (1.8)	0.15 (1.4)	0.19** (2.0)	0.20** (2.0)	0.20** (2.1)	0.20** (2.4)
No. obs.	1970	1335	1335	1366	1335	1366	1335	1366	1335	1366
F test	50 (0.0)	34 (0.0)	20 (0.0)	14 (0.0)	15 (0.0)	18 (0.0)	14 (0.0)	15 (0.0)	14 (0.0)	18 (0.0)
Hansen test	122 (85)	61 (57)	115 (117)	126 (117)	113 (117)	103 (117)	112 (117)	114 (117)	112 (117)	104 (117)
p -value	0.010	0.337	0.541	0.263	0.581	0.820	0.611	0.544	0.626	0.792
p -value of $H_0 : \pi_4 = 0$ for the $D_{i,t}^x = 1$ firms	n.a.	n.a.	0.01	0.02	0.09	0.10	0.07	0.01	0.05	0.00

wealth in Eq. (45), and column “2” also includes the observations with negative financial wealth using the broader definition in Eq. (48). The results show that the coefficient on $\ln w_{i,t-1}^F$ is positive, large in absolute value, and strongly significant for the likely constrained firms, and not significantly different from zero for the likely unconstrained firms. This result confirms the presence of financing constraints in the investment decisions of the firms that declare their cost of debt is too high in financing new investment projects. The last six columns report the results of the estimations that use the question about the lack of medium-to-long-term credit to select financially constrained firms. In this case the coefficient of $\ln w_{i,t-1}^F$ is again higher for the $D^{lc} = 1$ group than for the complementary sample, and is always very significant after we correct for the possible presence of distressed firms and firms that issue false reports (D_{-south}^{lc} and $D_{-s.\&lowyield}^{lc}$ columns). By adding the coefficient on $\ln w_{i,t-1}^F$ to the coefficient on $\ln w_{i,t-1}^F * D_{i,t}$, we obtain the wealth coefficient for the constrained firms. This ranges from 0.17 to 0.28 for the D^{hs} and $D_{-s.\&lowyield}^{lc}$ groups. These values are quite high compared to the same coefficient estimated for the constrained firms in the simulated industry (see Table 2). Simulation results in Table 4 show that the coefficient on $\ln w_{i,t-1}^F$ increases the tighter the collateral constraint (5) is. Therefore, the empirical results may indicate that physical capital has a low collateral value for the firms in the 1992 Mediocredito Survey.

The estimated coefficients on $\ln k_{i,t-1}$ and $\ln \theta_{i,t-1}$ do not differ significantly across the two groups of firms. The coefficient on $\ln n_{i,t}$ is lower for the financially constrained firms, even though its value is always positive and consistent with the restrictions of the structural model. The fact that the estimated coefficient on $\ln w_{i,t-1}^F$ is zero for the whole sample and for the groups of unconstrained firms allows us to use $\hat{\pi}_1$, $\hat{\pi}_2$, and $\hat{\pi}_3$ to estimate the structural parameters α , β , and γ using the restrictions in (44). The estimated $\hat{\alpha}$, $\hat{\beta}$, and $\hat{\gamma}$ are reported in Table 9. These are consistent with the values directly estimated from the production function (see Appendix E) and with the simple calculation of the elasticities using the factors' shares of output, which are reported at the top of Table 9. The fact that the restrictions imposed by the structural model on the coefficients on $\ln \theta_{i,t-1}$, $\ln k_{i,t-1}$, and $\ln n_{i,t}$ are not rejected by the estimation results is important because it confirms the validity of our structural model. Using the estimate of $\hat{\beta} = 0.502$, $\hat{\eta}$ equals 0.13 for the $D^{hs} = 1$ group. According to Eq. (24) this implies that if $w_{i,t}^F$ is 80% of $w_{i,t}^{MAX}$, then the shadow value of the binding borrowing constraint is equal to 2.9%. This value increases to 9.4% if $w_{i,t}^F$ is 50% of $w_{i,t}^{MAX}$.

In Table 10 we estimate Eq. (43) for the 1986–1991 sample, and we allow the coefficients to vary for the groups identified by the indirect criteria D^{age} , D^{divpol} and D^{size} . The coefficient on $\ln w_{i,t-1}^F$ is very small and not significant for all groups of likely unconstrained firms, while it is significantly positive for all groups of likely constrained firms except the $D^{divpol} = 1$ group. Regarding the other independent variables, the coefficients estimated for the likely unconstrained firms are always consistent with the restrictions of the structural model.

Among all the criteria used to split the sample, only the zero dividend policy has a limited ability to select firms with a higher correlation between variable investment and internal finance. This weak result may be due to an endogeneity problem in the selection criterion. Another possible explanation for this finding is that for privately owned firms, the zero dividend policy is not a very useful indicator of the intensity of financing constraints. This is because for many firms in the sample the controlling shareholders are

Table 9

Structural parameters of the production function; sample of 415 small and medium Italian manufacturing firms

The estimates $\tilde{\alpha}$, $\tilde{\gamma}$ and $\tilde{\beta}$ are computed as the following factor shares of output: $\tilde{\gamma}$ is the average of (labor cost/output), $\tilde{\beta}$ is the average of (materials cost/output), and $\tilde{\alpha}$ is the average of (user cost of fixed capital/output). The user cost of fixed capital is computed by assuming that $\delta = 0.1$. The estimates $\hat{\alpha}$, $\hat{\gamma}$ and $\hat{\beta}$ are derived from the corresponding columns of Table 8 for the groups of firms that are not likely to be financially constrained, using the restrictions in Eq. (44). D^{hs} is equal to one for firms that declare too high a cost of banking debt, and is equal to zero otherwise; D^{lc} is equal to one for firms with a lack of medium-to-long-term financing, and is equal to zero otherwise. In columns “2” financial wealth is defined by Eq. (48).

	1986–1991 sample	1988–1991 sample		
$\tilde{\alpha}$	0.040		0.043	
$\tilde{\gamma}$	0.229		0.230	
$\tilde{\beta}$	0.629		0.629	
		All firms	$D^{hs} = 0$ (2)	$D^{lc}_{-s.\&-lowyield} = 0$ (2)
$\hat{\alpha}$	0.154	0.108	0.025	0.092
$\hat{\gamma}$	0.319	0.420	0.492	0.272
$\hat{\beta}$	0.484	0.363	0.502	0.632

also the managers of the firms. These firms may choose zero dividends not because they are financially constrained, but because they have other ways of distributing revenues (such as in the form of compensation to managers) that are more tax efficient than dividends.

6. Robustness checks

Tables 8 and 10 show that the sensitivity of variable capital investment to internal finance is a useful indicator of the intensity of financing constraints. We argue that this finding is robust. First, this result is not likely to be driven by misspecification problems. That Eq. (43) is correctly specified is confirmed by the fact that we obtain plausible estimates of the structural parameters of the model. Second, our findings are robust to the possible criticism that the coefficient on $\ln w_{i,t-1}^F$ is positive because the productivity shock is measured with error. The analysis of the simulated data indicates that the coefficient on $\ln w_{i,t-1}^F$ is not affected by the measurement error in $\ln \theta_{i,t-1}$, because the two variables are uncorrelated. Our empirical results are consistent with this finding because the coefficient on $\ln w_{i,t-1}^F$ is always negative or not significantly different from zero, except for the group of likely financially constrained firms.

It therefore follows that the claim that the results are driven by measurement error in θ requires that: (i) $\ln \theta_{i,t-1}$ does not capture the unobservable productivity shock; and (ii) the unobservable productivity shock is highly correlated with $\ln w_{i,t-1}^F$ for likely constrained firms only, because on average these firms are more productive and grow faster than unconstrained firms. Assumption (i) is not very plausible, because the coefficient on $\ln \theta_{i,t-1}$ is significant and always has the expected sign and size for the likely unconstrained firms. Assumption (ii) is not plausible because Table 7 shows that firms that are more likely to be financially constrained have similar profitability as the other firms. These considerations indicate that the differences in the coefficient on $\ln w_{i,t-1}^F$ across groups are unlikely to be driven by unobservable investment opportunities.

Table 10

Financing constraints test based on the following reduced form variable capital equation: $\ln l_{i,t} = \pi_0 + a_i + d_t + \pi_1 \ln \theta_{i,t-1} + \pi_2 \ln k_{i,t-1} + \pi_3 \ln n_{i,t} + \pi_4 \ln w_{i,t-1}^F + \varepsilon_{i,t}$; sample of 415 small and medium Italian manufacturing firms, 1986 to 1991 period

In this table firms are selected according to indirect sorting criteria. In each “ D^x ” column, a dummy D^x , which identifies firms that are more likely to be financially constrained, is interacted with the constant and all the regressors. D^{age} is equal to one for firms founded after 1979 and is equal to zero otherwise; D^{divpol} is equal to one for firms with zero dividends in any period, and is equal to zero otherwise; and D^{size} is equal to one if for firm with less than 65 employees in 1992, and is equal to zero otherwise. *Significant at the 10% level; **significant at the 5% level; ***significant at the 1% level. The t -statistic is reported in parentheses. In columns “(1)” financial wealth is defined by Eq. (45). In columns “(2)” financial wealth is defined by Eq. (48). The coefficients are estimated with a two-step robust System GMM estimator (Blundell and Bond, 1998). The finite-sample correction to the two-step covariance matrix is derived by Windmeijer (2005). See the Legend to Table 8 for details.

	D^{age}		D^{divpol}		D^{size}	
	(1)	(2)	(1)	(2)	(1)	(2)
$\ln k_{i,t-1}$	0.33*** (3.4)	0.28** (2.2)	0.35** (3.3)	0.31** (2.3)	0.36** (3.1)	0.29** (2.0)
$\ln n_{i,t}$	0.59*** (4.7)	0.56*** (3.6)	0.42** (2.4)	0.29* (1.7)	0.46*** (2.6)	0.53*** (2.7)
$\ln \theta_{i,t-1}$	1.99* (1.7)	2.12* (1.6)	4.02*** (2.9)	4.50*** (2.6)	3.06*** (2.5)	3.27** (2.4)
$\ln w_{i,t-1}^F$	0.01 (0.2)	-0.08 (-1.5)	-0.01 (-0.1)	-0.09 (-1.3)	0.01 (0.2)	-0.03 (-0.6)
$\ln k_{i,t-1} * D_{i,t}$	-0.49** (-2.3)	-0.34* (-1.7)	-0.21 (-1.3)	-0.23 (-1.3)	-0.27 (-1.5)	-0.32 (-1.4)
$\ln n_{i,t} * D_{i,t}$	0.04 (0.2)	-0.09 (-0.5)	0.36* (1.6)	0.55** (2.5)	0.01 (0.1)	-0.43 (-1.4)
$\ln \theta_{i,t-1} * D_{i,t}$	-0.62 (-0.2)	0.56 (0.2)	-4.69** (-2.4)	-4.88** (-2.2)	-1.66 (-0.7)	-1.47 (-0.6)
$\ln w_{i,t-1}^F * D_{i,t}$	0.32** (2.2)	0.41*** (3.5)	0.09 (1.1)	0.29* (1.7)	0.24** (2.3)	0.26** (2.1)
Number of observations	1970	2017	1970	2017	1970	2017
F test	24 (0.0)	16 (0.0)	24 (0.0)	23 (0.0)	35 (0.0)	31 (0.0)
Hansen test	165 (175)	229 (175)	198 (175)	245 (175)	186 (175)	239 (175)
p -value	0.687	0.004	0.105	0.000	0.269	0.001
p -value of $H_0 : \pi_4 = 0$ for the $D_{i,t}^x = 1$ firms	0.02	0.002	0.15	0.21	0.01	0.040

Another possible criticism is that the $\ln w_{i,t-1}^F$ coefficient captures changes in the user cost of capital that are not related to financing constraints. By first differencing and introducing year dummy variables, we already take into account differences in the user costs of capital across firms or changes over time for all the firms. But one could object that the coefficient on $\ln w_{i,t-1}^F$ can be positive in the absence of financing imperfections if an increase in wealth is systematically correlated with a positive shock in the quality of the firm’s projects that also makes its investment less risky. We argue that it would be hard to justify such a systematic relationship. More importantly, if this is true then we should observe a positive

wealth coefficient for all firms, but this does not happen in our sample. The only possibility would then be that such a systematic relation only holds for likely financially constrained firms, because these are more risky or because they are younger firms for which the quality of management is very uncertain, and so their perceived riskiness is highly dependent on current performance. The results above reject both arguments. First, even though younger firms have a higher coefficient on $\ln w_{i,t-1}^F$, if we exclude these firms (the $D^{age} = 1$ observations) from the sample, we still obtain the same results illustrated in Tables 8 and 10.¹⁰ Second, likely financially constrained firms do not seem riskier, on average, than the other firms (see Table 7). Other robustness checks are illustrated in Sections 6.1 through 6.3 below.

6.1. Alternative definition of wealth

In this section we estimate Eq. (43) using a definition of wealth that does not include finished good inventories, denoted $\tilde{w}_{i,t-1}^F$. Table 11 shows the estimation results relative to the groups selected according to both the direct and the indirect criteria. The narrower definition of wealth implies that 16% of the observations with negative values of $\tilde{w}_{i,t-1}^F$ are not included. Most of these observations correspond to firms with low financial wealth that belong to the constrained groups (24% in the $D^{lc} = 1$ group and 23% in the $D^{hs} = 1$ group). This explains why the magnitude of the coefficient on $\ln \tilde{w}_{i,t-1}^F$ for these groups is much reduced. However, the results still largely confirm the findings of Tables 8 and 10. In particular, the coefficient on $\ln \tilde{w}_{i,t-1}^F$ is always negative and not significantly different from zero for the unconstrained sample, whereas is always larger for the constrained sample, and in three out of five cases significantly so.

6.2. Collateral value of the assets

The model developed in Section 2 assumes that fixed capital is the only physical collateral available to the firm. However, allowing variable capital to be collateral does not change the predictions of the model nor the interpretation of the results. If variable capital rather than fixed capital is used as the firm's collateral, then Eq. (5) becomes

$$\begin{aligned} b_{t+1} &\leq v_l l_{t+1}, \\ 0 < v_l &\leq 1. \end{aligned} \quad (49)$$

By substituting Eq. (49) (holding with equality) in the budget constraint (9), we obtain

$$d_t + \left(1 - \frac{v_l}{R}\right) l_{t+1} + k_{t+1} = w_t^F + (1 - \delta)k_{t-1}. \quad (50)$$

The larger v_l is, the smaller is the financial wealth needed to finance variable investment. This is equivalent to assuming that w^{\max} is smaller. Therefore, if v_l is sufficiently large then no firm is financially constrained and financial wealth should not be significant in Eq. (43) for both likely constrained and likely unconstrained firms. We find the opposite, however, confirming that v_l is relatively small in our sample. This finding is realistic because even though variable inputs are partly financed with trade credit, which is usually considered a

¹⁰Detailed results of the regressions performed after eliminating younger firms from the sample are available upon request.

Table 11

Financing constraints test based on the following reduced form variable capital equation: $\ln l_{i,t} = \pi_0 + a_i + d_t + \pi_1 \ln \theta_{i,t-1} + \pi_2 \ln k_{i,t-1} + \pi_3 \ln n_{i,t} + \pi_4 \ln \tilde{w}_{i,t-1} + \varepsilon_{i,t}$; sample of 415 small and medium Italian manufacturing firms, 1986–1991 period

In this table we use the alternative definition of wealth $\tilde{w}_{i,t}^F$, which is equal to $w_{i,t}^F$ (as defined in Eq. (45)) net of finished good inventories. In each “ D^x ” column, a dummy D^x , which identifies firms that are more likely to be financially constrained, is interacted with the constant and all the regressors. D^{hs} is equal to one for firms that declare too high a cost of banking debt, and is equal to zero otherwise; D^{lc} is equal to one for firms with a lack of medium-to-long-term financing, and is equal to zero otherwise; D^{age} is equal to one for firms founded after 1979 and is equal to zero otherwise; D^{divpol} is equal to one for firms with zero dividends in any period, and is equal to zero otherwise; and D^{size} is equal to one if for firm with less than 65 employees in 1992, and is equal to zero otherwise. *Significant at the 10% level; ** significant at the 5% level; ***significant at the 1% level. The t -statistic is reported in parentheses. The coefficients are estimated with a two-step robust System GMM estimator (Blundell and Bond, 1998). The finite-sample correction to the two-step covariance matrix is derived by Windmeijer (2005). See the legend to Table 8 for details.

	D^{hs}	D^{lc}	D^{age}	D^{divpol}	D^{size}
$\ln k_{i,t-1}$	−0.01 (−0.1)	0.16 (1.4)	0.28*** (2.6)	0.23** (1.9)	0.23* (1.8)
$\ln n_{i,t}$	1.06*** (6.4)	0.84*** (5.6)	0.64*** (4.6)	0.58*** (2.9)	0.77*** (4.9)
$\ln \theta_{i,t-1}$	1.27 (1.1)	1.48 (1.2)	0.77 (0.5)	3.23 (1.5)	3.03** (1.9)
$\ln \tilde{w}_{i,t-1}^F$	−0.02 (−0.8)	−0.04 (−1.3)	−0.05 (−1.6)	−0.07 (−1.4)	−0.05 (−1.5)
$\ln k_{i,t-1} * D_{i,t}$	0.37 (1.4)	0.21 (1.0)	−0.25* (−1.7)	−0.07 (−0.4)	−0.20 (−1.1)
$\ln n_{i,t} * D_{i,t}$	−0.54** (−1.9)	−0.25 (−1.3)	0.11 (0.8)	0.11 (0.5)	−0.59* (−1.9)
$\ln \theta_{i,t-1} * D_{i,t}$	−2.83 (−0.5)	4.32 (0.6)	1.92 (0.7)	−4.68 (−1.6)	−6.64** (−2.0)
$\ln \tilde{w}_{i,t-1}^F * D_{i,t}$	0.18* (1.8)	0.02 (0.2)	0.12 (1.5)	0.16* (1.8)	0.28*** (3.0)
Number of observations	1115	1115	1628	1628	1628
F test	17 (0.0)	21 (0.0)	22 (0.0)	27 (0.0)	29 (0.0)
Hansen test	114 (117)	99 (117)	193 (175)	237 (175)	215 (175)
p -value	0.545	0.878	0.159	0.001	0.021

form of collateralized debt, in practice trade credit is very costly. The annualized interest rate that firms implicitly pay on trade credit is often found to be above 40% (Ng, Smith, and Smith, 1999).

6.3. Estimations on the alternative data set

In this section we estimate Eq. (43) on the alternative sample based on the 1998 and 2001 Mediocredito Surveys. Each Survey asks the same type of questions about financing constraints, allowing us to pool them and obtain a larger sample. The disadvantage is that this alternative sample has less detailed balance sheet data: we do not have information

about plant and equipment separated from land and building, we do not have information about distributed and retained earnings, and we have a less detailed description of assets and liabilities.

Following the same procedure adopted for the main sample, we eliminate mergers and acquisitions and we include firms with at least eight years of balance sheet data, so as to have a complete set of instrumental variables in both surveys. Moreover, this sample also contains a small fraction of firms with less than 15 employees (2.1% of the total in this sample). The Employment Protection Law in Italy only applies to firms larger than 15 employees, and it imposes very high firing costs. Therefore, many very small firms decide not to grow above the 15-employee threshold in order to retain more flexibility (Schivardi and Torrini, 2004). This behavior distorts the relations among financing frictions, productivity shocks, and investment, and is likely to bias the results of our regressions. We therefore eliminate these firms from the sample, yielding an unbalanced panel of 964 firms and 7,305 observations.

In the finance section of the surveys, firms are asked the following questions (parentheses give the percentage of positive answers in the 1998 and 2001 surveys):

- (1) “During the last year, did the firm desire to borrow more at the interest rate prevailing on the market?” (13.5%, 19.3%).
- (2) “If the previous answer was yes: was the firm willing to pay a higher interest rate in order to get additional credit?” (5.0%, 6.9%).
- (3) “During the last year, did the firm ask for more credit without obtaining it?” (3.5%, 4.9%).

With respect to the questions in the 1992 Survey, these questions are less informative about the financing constraints faced by the firms in financing investment, as they are not specifically linked to the investment section. Another inconvenience is that these questions explicitly refer to only one sample year rather than to the entirety of the three years covered by the survey.

We find that question (3) is largely redundant, as few firms signal this problem and less than 0.5% of firms answer positively to question (3) without answering positively also to question (1). Question (2) is also a subset of question (1), but it may be able to identify a group of more financially constrained firms with a higher shadow value of money. Accordingly, we use questions (1) and (2) to construct the following dummies:

$D_i^{\text{rationed}} = 1$ if firm i answers positively to question (1) in either the 1998 or the 2001 Survey and $= 0$ otherwise.

$D_i^{\text{payhigher}} = 1$ if firm i answers positively to question (2) in either the 1998 or the 2001 Survey and $= 0$ otherwise.

The fraction of firms in the constrained groups is equal to 23.3% and 8.9% for D^{rationed} and $D^{\text{payhigher}}$, respectively. While we do not have information about dividend policy for these firms, we can construct the dummies relative to the size and age criteria: D^{age} identifies firms founded after 1982 (16% of all firms); D^{size} identifies firms with less than 25 employees (16% of all firms). For these indirect dummies we choose the thresholds such that the fraction of constrained firms equals the fraction in the age and size dummies used above. Table 12 illustrates the estimations of Eq. (43). We construct all variables following the same procedure adopted for the main sample, with two exceptions. First, we do not use the perpetual inventory method to compute the stock of fixed capital because the time

Table 12

Financing constraints test based on the following reduced form variable capital equation: $\ln l_{i,t} = \pi_0 + a_i + d_t + \pi_1 \ln \theta_{i,t-1} + \pi_2 \ln k_{i,t-1} + \pi_3 \ln n_{i,t} + \pi_4 \ln w_{i,t-1}^F + \varepsilon_{i,t}$; unbalanced panel of 964 small and medium Italian manufacturing firms, 1993–2000 period

In each “ D^x ” column, a dummy D^x , which identifies firms that are more likely to be financially constrained, is interacted with the constant and all the regressors. D^{rationed} is equal to one for firms that desire to borrow more at the interest rate prevailing on the market, and is equal to zero otherwise; $D^{\text{payhigher}}$ is equal to one for firms willing to pay a higher interest rate in order to get additional credit, and is equal to zero otherwise; D^{age} is equal to one for firms founded after 1982 and is equal to zero otherwise; D^{size} is equal to one if for firm with less than 25 employees in 2001, and is equal to zero otherwise. *Significant at the 10% level; **significant at the 5% level, ***significant at the 1% level. The t -statistic is reported in parentheses. In columns “(1)” financial wealth is defined by Eq. (45). In columns “(2)” financial wealth is defined by Eq. (48). The coefficients are estimated with a two-step robust System GMM estimator (Blundell and Bond, 1998). The finite-sample correction to the two-step covariance matrix is derived by Windmeijer (2005). See the legend to Table 8 for details.

	D^{rationed}		$D^{\text{payhigher}}$		D^{age}		D^{size}	
	(1)	(2)	(1)	(2)	(1)	(2)	(1)	(2)
$\ln k_{i,t-1}$	0.19*** (3.4)	0.22*** (4.4)	0.20*** (3.9)	0.21*** (4.6)	0.21*** (3.6)	0.22*** (4.1)	0.20*** (3.6)	0.20*** (4.0)
$\ln n_{i,t}$	0.75*** (9.7)	0.80*** (10.9)	0.73*** (10.2)	0.79*** (11.0)	0.68*** (8.9)	0.68*** (9.5)	0.72*** (8.9)	0.72*** (8.9)
$\ln \theta_{i,t-1}$	1.40*** (3.6)	1.43*** (3.6)	1.45*** (3.6)	1.26*** (3.1)	1.60*** (4.0)	1.51*** (4.1)	1.47*** (4.0)	1.26*** (3.6)
$\ln w_{i,t-1}^F$	-0.04 (-1.4)	-0.05 (-1.5)	-0.01 (-0.3)	-0.03 (-1.2)	-0.03 (-0.9)	-0.03 (-1.1)	-0.02 (-0.6)	-0.05 (-1.4)
$\ln k_{i,t-1} * D_{i,t}$	-0.08 (-0.8)	-0.09 (-0.8)	-0.06 (-0.4)	-0.12 (-0.6)	-0.08 (-0.9)	-0.01 (-0.1)	0.00 (0.0)	0.03 (0.3)
$\ln n_{i,t} * D_{i,t}$	-0.07 (-0.6)	-0.11 (-0.7)	-0.06 (-0.3)	-0.15 (-0.7)	0.06 (0.6)	-0.03 (-0.3)	-0.34* (-1.8)	-0.25** (-2.0)
$\ln \theta_{i,t-1} * D_{i,t}$	-1.22 (-1.4)	-1.37* (-1.6)	-0.81 (-0.9)	-1.02 (-1.2)	-1.89 (-1.5)	-2.23** (-2.1)	-0.82 (-0.7)	-0.60 (-0.5)
$\ln w_{i,t-1}^F * D_{i,t}$	0.15*** (2.7)	0.18*** (2.8)	0.14* (1.6)	0.26*** (2.8)	0.01 (0.2)	0.05 (0.7)	0.07 (0.8)	0.16** (2.5)
No. obs.	4656	5266	4656	5266	4656	5266	4656	5266
F test	24 (0.0)	36 (0.0)	34 (0.0)	40 (0.0)	31 (0.0)	26 (0.0)	39 (0.0)	40 (0.0)
Hansen test	289 (254)	285 (254)	206 (254)	219 (254)	267 (254)	287 (254)	258 (254)	278 (254)
p -value	0.089	0.086	0.988	0.945	0.248	0.076	0.411	0.141
p -value of $H_0 : \pi_4 = 0$ for the $D_{i,t}^x = 1$ firms	0.019	0.019	0.123	0.015	0.750	0.076	0.497	0.044

series is too short for most of the firms in the sample. Instead, we evaluate fixed capital at book value. Moreover, we do not subtract the dividend payments for the calculation of $w_{i,t-1}^F$ because we do not have this information for this sample.

The results obtained for the D^{rationed} and $D^{\text{payhigher}}$ dummies confirm the validity of the financing constraints test. The coefficient on $\ln w_{i,t-1}^F$ is not significantly different from zero for the likely unconstrained firms and is always positive and significantly higher for the likely constrained firms. Moreover, the estimate of the coefficient on $\ln w_{i,t-1}^F$ for the firms that answer yes to both questions 1 and 2 (the sum of the coefficients on $\ln w_{i,t-1}^F$ and on

$\ln w_{i,t-1}^F * D_{i,t}^{payhigher}$) is higher than for the firms that answer yes only to question 1, as predicted by the model. The estimated coefficients on $\ln \theta_{i,t-1}$, $\ln k_{i,t-1}$, and $\ln n_{i,t}$ are always strongly significant for the likely unconstrained firms and do not show significant deviations for the likely constrained firms.

The coefficient on $\ln w_{i,t-1}^F$ is between 0.17 and 0.28 in the panel based on the 1992 Survey, but the same coefficient is between 0.11 and 0.23 for the alternative panel of firms based on the 1998 and 2001 surveys. Further, in the alternative panel the coefficient on $\ln w_{i,t-1}^F$ is always higher for younger and smaller firms than for the complementary samples, but the difference is significant in only one out of four cases.

Because in both panels the estimates for the likely unconstrained firms are consistent with the predictions of the model, we can interpret these findings as evidence that the intensity of financing constraints on firm investment was lower in the 1995–2000 period than in the 1988 to 1991 period. This may be due to the increased efficiency of the Italian financial sector after 1992, driven by the liberalization of financial services in the Euro area. For example, Jappelli and Pistaferri (2000) note that until the beginning of the 1990s the average down payment ratio in Italy for mortgaged debt was usually between 40% and 50%, as opposed to 20% in the US and 15% in the U.K. However, the same authors show that since 1994, Italian banks have offered mortgages with down payment requirements as low as 20%, in response to increased internal and international competition. This may have reduced considerably the financing frictions faced by small and young Italian firms.

7. Conclusions

In this paper we develop a new test to detect financing constraints on firm investment. The test is derived from a structural multifactor model of firm investment with financing imperfections and is based on a reduced-form variable capital investment equation.

We solve the model using a numerical method and we simulate two industries, one with quadratic adjustment costs, the other with the irreversibility of fixed capital. Both industries are calibrated to match the U.S. industry. The results of the simulations show that the correlation between variable capital investment and internal finance is a useful indicator of the intensity of financing constraints, even when firm investment opportunities are estimated with error and regardless of the type of adjustment costs of fixed capital.

We verify the validity of this test on two samples of Italian manufacturing firms. First, the estimation results do not reject the restrictions imposed by the structural parameters. Second, the sensitivity of variable investment to internal finance is never significant and is always very small for the groups of firms a priori not expected to be financially constrained. By contrast, it is significantly greater than zero and often large for firms that are likely to be financially constrained. The fact that the reduced-form parameters do not reject the restrictions imposed by the structural model implies that we can interpret the magnitude of this sensitivity as an indicator of the intensity of financing constraints.

One important property of this test is that it does not require information about the market value of the firm. Because it requires only the information present in balance sheet data, it can be easily applied to small privately owned firms not quoted on the stock markets. This property is useful for the literature that studies the consequences of financing imperfections for aggregate fluctuations, such as the literature on the financial accelerator and on the credit channel of monetary policy.

Appendix A

In order to prove that a solution to the firm's investment problem exists and is unique, it is helpful to define the value function in (11) as follows:

$$V_t(a_t, \theta_t, k_t, l_t) = \underset{k_{t+1}, l_{t+1}, a_{t+1}}{MAX} \pi_t + \frac{\gamma}{R} E_t[V_{t+1}(a_{t+1}, \theta_{t+1}, k_{t+1}, l_{t+1})]. \quad (51)$$

We use Eqs. (1), (7), (8), and (9) to rewrite π_t as a function of the state variables in periods t and $t + 1$:

$$\pi_t = \theta_t k_t^\alpha l_t^\beta + a_t + (1 - \delta)k_t - \frac{a_{t+1}}{R} - k_{t+1} - l_{t+1}. \quad (52)$$

The return function π_t is real valued and continuous. Moreover, it is bounded, because the production function is concave, the productivity shock θ_t is a stationary process, and Assumption 1 ensures that a_t is bounded. Finally, Eq. (52) proves that π_t is strictly increasing in the state variables at time t . Since constraints (5), (6), and (9) define a compact and convex feasibility set for the choice variables l_{t+1} , k_{t+1} , b_{t+1} , and d_t , it follows that the model satisfies the conditions for Theorems 9.6 and 9.8 in Stokey and Lucas (1989), ensuring that the solution to the problem exists and is unique.

Appendix B. Proof of Proposition 1

Let w_t^{\max} be the level of financial wealth that allows the financing of all profitable investment projects. If $w_t^F \geq w_t^{\max}$, then $l_{t+1} = l_{t+1}^*(\theta_t, k_t)$ and $k_{t+1} = k_{t+1}^*(\theta_t, k_t)$. It follows that w_t^{\max} satisfies the condition

$$\left(1 - \frac{v}{R}\right) k_{t+1}^* + l_{t+1}^* = w_t^{\max} + (1 - \delta)k_t. \quad (53)$$

Suppose now that w_t^F decreases below w_t^{\max} . Eq. (53) cannot be satisfied with equality.

If the irreversibility constraint is binding with equality, then $k_{t+1} = (1 - \delta)k_t$. In this case a reduction of w_t^F causes a reduction in l_{t+1} below l_{t+1}^* . The proof of Proposition 1 follows by the fact that the production function (1) implies that $E_t\left(\frac{\partial y_{t+1}}{\partial l_{t+1}}\right)$ is decreasing and concave in l_{t+1} conditional on $k_{t+1} = (1 - \delta)k_t$.

If the irreversibility constraint is not binding, then both l_{t+1} and k_{t+1} must decrease as w_t^F decreases below w_t^{\max} , because the two factors of production are complementary. This still implies that $E_t\left(\frac{\partial y_{t+1}}{\partial l_{t+1}}\right)$ is decreasing and concave in w_t^F , because the production function is concave in both factors.

Appendix C

We briefly describe the method we use to solve the dynamic maximization problem of the firm. We discretize the state space of w_t , k_t and θ_t into 20 grid points for each variable. We model θ^I as a two-state i.i.d. process. We guess the value function $E_t[V_{t+1}(w_{t+1}, \theta_{t+1}, k_{t+1})]$, and based on this guess we find the policy functions $k_{t+1}(w_t, \theta_t, k_t)$, $l_{t+1}(w_t, \theta_t, k_t)$, and $b_{t+1}(w_t, \theta_t, k_t)$ that maximize $V_t(w_t, \theta_t, k_t)$. We use the

maximized value function to reformulate a guess of $E_t[V_{t+1}(w_{t+1}, \theta_{t+1}, k_{t+1})]$, and we repeat this procedure until convergence is achieved.

Appendix D

We describe here the variables used in the empirical analysis of the paper:

$p_t^y y_{i,t}$: total revenues realized during year t , at current prices.

$p_t^k k_{i,t}$: sum of the replacement value of (i) plants and equipment, and (ii) intangible fixed capital (Software, Advertising, Research and Development). We include in $p_t^k k_{i,t}$ all capital purchased before the end of time t . We compute the replacement value of capital by adopting the following perpetual inventory method:

$$p_{t+1}^{kj} k_{i,t+1}^j = p_t^{kj} k_{i,t}^j (1 + \pi_t^j) (1 - \delta^j) + p_{t+1}^{kj} i_{i,t+1}^j,$$

$j = \{1, 2\}$, where 1 = plant and equipment, and 2 = intangible fixed capital. $\pi^1 = \%$ change in the producer price index for agricultural and industrial machinery (source: OECD, from Datastream); $\pi^2 = \%$ change in the producer price index (source: OECD, from Datastream). δ^j are estimated separately for the 20 manufacturing sectors using aggregate annual data on the replacement value and the total depreciation of the capital (source: ISTAT, the Italian National Statistical Institute). Given that within each sector depreciation rates vary only marginally between years, we conveniently use the average over the sample period: δ^1 ranges from 9.3% to 10.7%, and δ^2 from 8.4% to 10.6%.

$p_t^l l_{i,t}$: this variable measures the use of variable inputs, at current prices, and is computed as follows: beginning-of-period t input inventories (materials and work in progress), plus new purchases of materials in period t , minus end-of-period t input inventories.

$p_t^n n_{i,t}$: this variable includes the total cost of the labor in year t , at current prices.

$p_t^w w_{i,t}^F$: operative profits during period $t - 1$ (value of production minus the cost of production inputs) plus net short-term financial assets (after dividend payments) and the stock of finished good inventories at the beginning of period $t - 1$ multiplied by one plus the nominal interest rate.

$p_t^w \tilde{w}_{i,t}^F$: equals $p_t^w w_{i,t}^F$ minus the stock of finished good inventories.

In order to transform the variables into real terms, we use the following price indexes (source: ISTAT, the Italian National Statistical Institute):

p_t^y : consumer price index relative to all products excluding services.

p_t^w : same as p_t^y .

p_t^k : producer price index of durable inputs.

p_t^n : wage earnings index of the manufacturing sector.

p_t^l : wholesale price index for intermediate goods.

Appendix E

In this section we illustrate the procedure used to estimate the productivity shock $\ln \theta_{i,t}$. First, we directly estimate the output elasticities to factor inputs α , β , and γ . We consider the production function in Eq. (41). Table 13 reports summary statistics of $y_{i,t}$, $k_{i,t}$, $l_{i,t}$, and

$n_{i,t}$. By taking logs, we have the following linearized version of Eq. (41):

$$\ln y_{i,t} = a_i + d_t + x_{s,t} + \alpha \ln k_{i,t-1} + \beta \ln l_{i,t} + \gamma \ln n_{i,t} + \varepsilon_{i,t}, \quad (54)$$

where a_i is the firm fixed effect, d_t is the time effect and, $x_{s,t}$ is the sector effect (we consider two-digit sectors as classified by ISTAT). In order to allow for some heterogeneity in the technology employed by firms in different sectors, Eq. (54) is separately estimated for seven groups of firms. Each group consists of firms with production activity that is as homogeneous as possible. Table 14 shows the composition of the groups. Because we estimate Eq. (41) also for those firms that split or merged during the sample period, the total number is 561 firms. Eq. (54) is estimated by first differencing and then using GMM with instrumental variables on the sample from 1985 to 1992. We use both lagged first differences and levels as instruments for the equation in first differences. We consider lags -1 and -2 . This means that we exclude year 1982 in order to diminish possible distortions caused by the perpetual inventory method, and we have the data from 1983 and 1984 available as instruments. Table 15 reports estimation results. The first column is relative to the whole sample, while the next seven columns show the estimates of $\hat{\alpha}$, $\hat{\beta}$, and $\hat{\gamma}$ for the seven groups separately. The Wald test shows that the restriction $\hat{\alpha} + \hat{\beta} + \hat{\gamma} = 1$ is rejected in favor of $\hat{\alpha} + \hat{\beta} + \hat{\gamma} < 1$ for all groups except group 7. The estimated output elasticity of variable capital $\hat{\beta}$ ranges between 0.29 and 0.56, and in three groups it is higher than the output elasticity of labor $\hat{\gamma}$. These high estimates of β are quite common in firm-level estimates of the production function (see, for example, Hall and Mairesse, 1996). Output

Table 13

Summary statistics of the variables used to estimate the production function

Values are in billions of Italian Lire, 1982 prices. 1 billion lire was equal to 0.71 million US\$ at the 1982 exchange rate. $y_{i,t}$ is total revenues; $k_{i,t}$ is the replacement value of the plant, equipment and other intangible fixed assets; $l_{i,t}$ is the usage of materials; and $n_{i,t}$ is labor cost.

Variable	Mean	St. dev	Min	Max
$y_{i,t}$	33.105	68.002	1.095	1162.078
$l_{i,t}$	19.582	51.121	0.093	1200.405
$n_{i,t}$	11.303	19.475	0.343	235.296
$k_{i,t}$	8.179	18.454	0.067	259.543

Table 14

Composition of the groups for which the production function is separately estimated

Two Digits ISTAT ^a sectors	Number of firms
Group 1: Industrial Machinery	78
Group 2: Electronic Machinery, Precision Instruments	49
Group 3: Textiles, Shoes and Clothes, Wood Furniture	117
Group 4: Chemicals, Rubber and Plastics	63
Group 5: Metallic Products	80
Group 6: Food, Sugar and Tobaccos, Paper and Printing	66
Group 7: Non-metallic Minerals, Other Manufacturing	108

^aItalian National Statistic Institute.

Table 15

Production function estimation results; balanced panel of 561 firms, 1985–1992 period

* One coefficient relative to a two-digit sector dummy variable is estimated here. ** Only $t - 1$ instruments are used for the estimation of this group, due to the reduced number of observations. *** Wald test of the following restriction: $\alpha + \beta + \gamma = 1$. Standard deviations are in parentheses. $\hat{\alpha}$ is the estimated elasticity of output to fixed capital, $\hat{\beta}$ is the estimated elasticity of output to variable capital, and $\hat{\gamma}$ is the estimated elasticity of output to labor. Sargan test is a test of the overidentifying restrictions.

	All firms	Group 1	Group 2	Group 3	Group 4	Group 5	Group 6	Group 7
$\hat{\alpha}$	0.111 (0.02)	0.105 (0.02)	0.062 (0.015)	0.114 (0.03)	0.081 (0.02)	0.038 (0.022)	0.040 (0.01)	0.198 (0.02)
$\hat{\beta}$	0.389 (0.02)	0.377 (0.01)	0.289 (0.013)	0.424 (0.03)	0.454 (0.01)	0.393 (0.017)	0.562 (0.01)	0.406 (0.024)
$\hat{\gamma}$	0.441 (0.03)	0.494 (0.02)	0.468 (0.023)	0.348 (0.04)	0.193 (0.01)	0.491 (0.034)	0.350 (0.01)	0.401 (0.05)
Sargan test	65.50	38.90	25.78	39.87	39.71	38.20	45.18	33.64
D.f.	37	37	27**	37	37	37	36*	36*
p -value	0.00	0.38	0.53	0.34	0.35	0.40	0.14	0.58
χ^2 ***		29.7	41.7	814.6	217.2	9.61	11.35	0.01
p -value		0.00	0.00	0.00	0.00	0.00	0.00	0.91
No. firms	561	78	49	117	63	80	66	108
No. obs.	4488	624	392	936	504	640	528	864

elasticity of fixed capital $\hat{\alpha}$ ranges between 0.04 and 0.11. This range of values is reasonable and consistent with the factor shares of output, given the amount of fixed capital as opposed to variable capital used in the production (see Tables 9 and 13), and the difference in the user costs of fixed and variable capital caused by the difference in the depreciation factors. The yearly depreciation rate of plant and equipment is around 10%, while the depreciation rate of the use of materials is by construction equal to 100%. The overidentifying restrictions are rejected for the estimation of the whole sample, but not for the estimations for each group of firms. Using the estimated elasticities $\hat{\alpha}$, $\hat{\beta}$, and $\hat{\gamma}$, we compute total factor productivity for all the firm-year observations:

$$\widehat{TFT}_{i,t} = \ln y_t - \hat{\alpha} \ln k_{i,t-1} + \hat{\beta} \ln l_{i,t} - \hat{\gamma} \ln n_{i,t}. \quad (55)$$

We then regress $\widehat{TFT}_{i,t}$ on fixed effects and year and sector dummy variables. The estimated residual from this regression is $\ln \theta_{i,t}$, which is the estimated productivity shock at the beginning of period $t + 1$.

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